

Matthie Recalculation Path

Proportional Reasoning (Book 3) Teacher Guide & Keys

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Built against Master Framework v1.1

Teacher Guide & Keys

This guide covers every skill in the Proportional Reasoning skill map (46 skills across Chapters 1-16), plus the chapter-level support material (materials, differentiation, and enrichment) that doesn't attach to one specific skill. Each skill entry gives the mathematical claim, a Route Finder item, a Tripwire item, a Confirmation item in a different surface form, a Recalculation Check, and a Road Test -- plus the full teacher pathway: observed response, likely reasoning (a hypothesis, not a diagnosis), how to confirm it, the teaching move, and where to route next.

A single wrong answer is a hypothesis, not a diagnosis. Confirm with a second independent piece of evidence before treating a pattern as established, per the confirmation rule.

Teacher Desk Card

Keep this page open while working through any skill below. Four rules, one per stage of the cycle:

FIND: STOP FINDING when the evidence is sufficient to choose a teaching target. Do not measure everything the student cannot do.

TEACH: STOP RETEACHING the wider unit when the confirmed gap is narrower. Address the smallest idea actually blocking progress.

CHECK: STOP TESTING when the student has provided sufficient independent evidence. A fresh problem or existing classwork both count.

MOVE: STOP WORKING THE OLD TARGET once the student has met the defined mastery criterion. Record status and route to what's next.

Status codes: NY (Not Yet) -- R (Recalculating) -- M (Mastered) -- IE (Insufficient Evidence). Fixed across the whole series; never renamed per book.

The lightest-evidence rule: do not administer a separate diagnostic form when reliable, current classwork already answers the question.

Discussion and Evidence Protocol

Applies book-wide, every chapter. Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Quick Answer Key (Student Forms)

For fast grading against a completed Student Form. Organized in the same order as Student Forms -- Route Finder, Tripwire, Confirmation, Road Test, per skill. For the reasoning, the reteach move, and where to route a student next, see that skill's full entry later in this guide. Strategy Studio tasks are open response by design and aren't included here -- judge those against the chapter's own Strategy Studio framing.

Form ID	Item	Correct response
PR-RR-01-RF1	Route Finder	Yes. $4 \div 1 = 4$, $8 \div 2 = 4$, $12 \div 3 = 4$ -- constant quotient, and (0,0) fits.
PR-RR-01-RF1	Tripwire	No. The difference is constant (\$2 per item) but the quotient is not ($7 \div 2 = 3.5$, $11 \div 4 = 2.75$), and the cost at 0 items is \$3, not \$0.
PR-RR-01-CF1	Confirmation	The no-fee gym only. Its cost at 0 visits is \$0 and the quotient is constant at 5; the membership gym's cost at 0 visits is \$10, not \$0.
PR-RR-01-CF1	Road Test	Yes -- $9 \div 2 = 4.50$ and $22.50 \div 5 = 4.50$, constant quotient, and \$0 for 0 packages is consistent with the rate.
PR-RR-02-RF1	Route Finder	\$21 (unit rate \$3/notebook, then $7 \times \$3$).
PR-RR-02-RF1	Tripwire	\$15 (unit rate \$1.50/pencil, $10 \times \$1.50$).
PR-RR-02-CF1	Confirmation	Find the time for 1 lap first ($12 \div 3 = 4$ minutes), then multiply by 7 (28 minutes) -- describes the unit-first method, not an additive shortcut.
PR-RR-02-CF1	Road Test	1350 mL ($150 \text{ mL/serving} \times 9$).
PR-RR-03-RF1	Route Finder	Any correct unit-rate sentence, e.g. "6 is the cost in dollars for every 1 item."
PR-RR-03-RF1	Tripwire	A stated sentence such as "4.5 means \$4.50 per item" or "4.5 hours per trip," with units.
PR-RR-03-CF1	Confirmation	Any correct sentence, e.g. "7 km per hour -- the runner covers 7 km every hour."

Form ID	Item	Correct response
PR-RR-03-CF1	Road Test	3 liters per minute -- the tank fills 3 liters every minute, found by dividing 9 by 3.
PR-FR-01-RF1	Route Finder	No -- 15 students vs. 21 students. Same fraction, different wholes.
PR-FR-01-RF1	Tripwire	No, not necessarily -- the fraction is the same but the wholes (total class sizes) may differ.
PR-FR-01-CF1	Confirmation	No -- about 27 teachers vs. 400 students. Same fraction, very different wholes.
PR-FR-01-CF1	Road Test	No -- the number of oak trees depends on each park's total number of trees, which may differ.
PR-FR-02-RF1	Route Finder	12 ($\frac{1}{5}$ of 20 = 4, then $3 \times 4 = 12$).
PR-FR-02-RF1	Tripwire	20 ($\frac{1}{6}$ of 24 = 4, then $5 \times 4 = 20$).
PR-FR-02-CF1	Confirmation	\$14 -- find $\frac{1}{9}$ of \$63 first (\$7), then multiply by 2 (\$14).
PR-FR-02-CF1	Road Test	30 km ($\frac{1}{8}$ of 48 = 6, then $5 \times 6 = 30$).
PR-FR-03-RF1	Route Finder	$\frac{5}{12}$ is greater (common denominator 24: $\frac{9}{24}$ vs. $\frac{10}{24}$).
PR-FR-03-RF1	Tripwire	$\frac{2}{5}$ is actually greater (common denominator 40: $\frac{16}{40}$ vs. $\frac{15}{40}$) -- the digit-comparison shortcut gives the wrong answer here, which is exactly why the conversion step is required every time, not just when convenient.

Form ID	Item	Correct response
PR-FR-03-CF1	Confirmation	$\frac{7}{12}$ cup (common denominator 36: $\frac{20}{36}$ vs. $\frac{21}{36}$).
PR-FR-03-CF1	Road Test	The $\frac{4}{7}$ container has more room left (it's less full: $\frac{4}{7} = \frac{20}{35}$, $\frac{3}{5} = \frac{21}{35}$, so $\frac{3}{5}$ is fuller, leaving less room).
PR-FR-04-RF1	Route Finder	$\frac{10}{3}$ cups, or 3 and $\frac{1}{3}$ cups ($\frac{2}{3} \times 5$).
PR-FR-04-RF1	Tripwire	$\frac{1}{2}$ cup ($\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$), smaller than the original $\frac{2}{3}$ cup, which makes sense since a fraction less than 1 was applied.
PR-FR-04-CF1	Confirmation	$\frac{5}{9}$ of a can ($\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$).
PR-FR-04-CF1	Road Test	$\frac{3}{5}$ of an acre ($\frac{4}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5}$).
PR-FR-05-RF1	Route Finder	$\frac{7}{12}$ (common denominator 12: $\frac{4}{12} + \frac{3}{12}$).
PR-FR-05-RF1	Tripwire	$\frac{11}{15}$ (common denominator 15: $\frac{6}{15} + \frac{5}{15}$).
PR-FR-05-CF1	Confirmation	$\frac{11}{12}$ (common denominator 12: $\frac{8}{12} + \frac{3}{12}$).
PR-FR-05-CF1	Road Test	$\frac{11}{20}$ of a gallon (common denominator 20: $\frac{6}{20} + \frac{5}{20}$).
PR-FR-06-RF1	Route Finder	8 servings ($6 \div \frac{3}{4} = 8$).

Form ID	Item	Correct response
PR-FR-06-RF1	Tripwire	$8 (4 \div \frac{1}{2} = 8$ -- how many halves fit into 4).
PR-FR-06-CF1	Confirmation	15 pieces ($9 \div \frac{3}{5} = 15$).
PR-FR-06-CF1	Road Test	12 bottles ($10 \div \frac{5}{6} = 12$).
PR-PC-01-RF1	Route Finder	$0.7 = 70\%$; $7\% = 0.07$. Not the same value -- this pair tests the tenths/hundredths distinction directly.
PR-PC-01-RF1	Tripwire	60% ($0.6 = \frac{60}{100}$).
PR-PC-01-CF1	Confirmation	9%.
PR-PC-01-CF1	Road Test	30%.
PR-PC-02-RF1	Route Finder	225% ($\frac{45}{20} = 2.25 = 225\%$).
PR-PC-02-RF1	Tripwire	225% ($\frac{90}{40} = 2.25$).
PR-PC-02-CF1	Confirmation	135% ($\frac{54}{40} = 1.35$).
PR-PC-02-CF1	Road Test	250% of the original ($\frac{5000}{2000} = 2.5$).
PR-PC-03-RF1	Route Finder	Any correctly justified common form (e.g., convert both to percent: 25% vs. 30%) with a stated reason.
PR-PC-03-RF1	Tripwire	Either can be correct depending on purpose; the answer must discuss why (e.g., percent for comparing across different totals, fraction for exact part-counting), not declare one form universally right.
PR-PC-03-CF1	Confirmation	Any reasonably justified answer (fraction $\frac{3}{8}$ keeps exactness; decimal 0.375 is easier to multiply on a calculator).

Form ID	Item	Correct response
PR-PC-03-CF1	Road Test	Both describe the same proportion ($25\% = \frac{1}{4}$); which is clearer depends on the audience and purpose.
PR-PC-04-RF1	Route Finder	10 students ($40\% = \frac{40}{100}$, $\frac{40}{100} \times 25 = 10$).
PR-PC-04-RF1	Tripwire	12 people ($30\% = 0.30$, $0.30 \times 40 = 12$; the whole is 40 people, not 100).
PR-PC-04-CF1	Confirmation	90 students (0.60×150).
PR-PC-04-CF1	Road Test	12 cookies (0.15×80).
PR-PC-05-RF1	Route Finder	25%, because 23 is closer to 25 than to 10 or 50 by actual distance.
PR-PC-05-RF1	Tripwire	50%, because 48 is closer to 50 by actual distance.
PR-PC-05-CF1	Confirmation	75%, because 63 is 12 percentage points from 75% and 13 percentage points from 50%.
PR-PC-05-CF1	Road Test	Nearest 25%-benchmark is 75% (distance 2), closer than 100% (distance 23).
PR-PC-06-RF1	Route Finder	Change: \$10. Final: \$60 (120% of \$50).
PR-PC-06-RF1	Tripwire	Change: \$12. Final: \$92.
PR-PC-06-CF1	Confirmation	Change: \$300. Final: \$800.
PR-PC-06-CF1	Road Test	Change: 120 employees (60% increase). Final: 320, which is 160% of the original 200.
PR-PC-07-RF1	Route Finder	The percent is missing (part=15 and whole=60 are both given).
PR-PC-07-RF1	Tripwire	The whole is missing (part=20, percent=25% are given); correct approach is $20 \div 0.25 = 80$.
PR-PC-07-CF1	Confirmation	The whole is missing (part=45, percent=60% are given).
PR-PC-07-CF1	Road Test	The whole (total reviewers) is missing; part=9, percent=30%.
PR-PC-08-RF1	Route Finder	60 ($24 \div 0.40 = 60$ -- dividing, since the whole is missing).
PR-PC-08-RF1	Tripwire	For “find the whole” or “find the percent,” division is required, not multiplication.

Form ID	Item	Correct response
PR-PC-08-CF1	Confirmation	80 ($36 \div 0.45$).
PR-PC-08-CF1	Road Test	30% removed ($\frac{75}{250} = 0.30$).
PR-PC-09-RF1	Route Finder	$\frac{20}{80} = 0.25 = 25\%$. Confirmed.
PR-PC-09-RF1	Tripwire	Verification should substitute back into the relationship (part = percent $\times$ whole: $0.25 \times 72 = 18$ confirms it), not repeat the identical calculation.
PR-PC-09-CF1	Confirmation	Not fully independent -- repeating the same division can't catch an error made in that same division. A substitution-style check ($0.40 \times 80 = 32$) or a different method is needed.
PR-PC-09-CF1	Road Test	Yes -- substituting back, $0.30 \times 70 = 21$, confirmed.
PR-PC-10-RF1	Route Finder	\$30 (25% of \$40 = \$10; \$40 - \$10 = \$30).
PR-PC-10-RF1	Tripwire	\$42 (30% of \$60 = \$18; \$60 - \$18 = \$42).
PR-PC-10-CF1	Confirmation	No -- 15% of \$90 is \$13.50, not \$15; the sale price is \$76.50.
PR-PC-10-CF1	Road Test	\$70.20 (\$65 + \$5.20).
PR-PC-11-RF1	Route Finder	Increases; \$230.
PR-PC-11-RF1	Tripwire	\$55 (markup adds; \$50 + \$5).
PR-PC-11-CF1	Confirmation	Down; \$114.
PR-PC-11-CF1	Road Test	Increases -- interest is always added to a balance.
PR-PC-12-RF1	Route Finder	\$40 both ways: (1) \$10 off, $50 - 10 = 40$; (2) $50 \times 0.80 = 40$.
PR-PC-12-RF1	Tripwire	Correct answer is \$68 (80×0.85); the student should notice and resolve the discrepancy, not just pick one.
PR-PC-12-CF1	Confirmation	\$224 both ways.
PR-PC-12-CF1	Road Test	\$49.50 is correct (45×1.10); the mismatch shows one method has an error that must be found before trusting either answer.
PR-PC-13-RF1	Route Finder	\$86.40 ($100 \times 0.80 = 80$, then $80 \times 1.08 = 86.40$).

Form ID	Item	Correct response
PR-PC-13-RF1	Tripwire	\$86.40, taxing the \$80 sale price, not the original \$100.
PR-PC-13-CF1	Confirmation	\$225.25 ($\$250 \times 0.85 = \$212.50$, then $\times 1.06 = \$225.25$).
PR-PC-13-CF1	Road Test	Final price is 126% of wholesale cost ($1.40 \times 0.90 = 1.26$), so the base at each step must be tracked, not the original wholesale cost reused.
PR-PC-14-RF1	Route Finder	No -- lower. $\$100 \rightarrow \$120 \rightarrow \$96$, not back to \$100.
PR-PC-14-RF1	Tripwire	No -- $\$100 \rightarrow \$125 \rightarrow \$93.75$, lower than the original \$100.
PR-PC-14-CF1	Confirmation	No -- lower. $\$100 \rightarrow \$70 \rightarrow \$91$, not \$100.
PR-PC-14-CF1	Road Test	Not back to even -- $\$100 \rightarrow \$50 \rightarrow \$75$, a net loss of 25%.
PR-PC-15-RF1	Route Finder	\$400 ($\$500 \div 1.25 = \$400$).
PR-PC-15-RF1	Tripwire	\$200 ($\$260 \div 1.30 = \$200$).
PR-PC-15-CF1	Confirmation	\$120 ($1000 \times 0.04 \times 3$ -- same rate on the same original principal each year).
PR-PC-15-CF1	Road Test	\$150 ($\$172.50 \div 1.15$).
PR-RR-04-RF1	Route Finder	$\frac{3}{8}$ (whole = $3+5 = 8$ parts).
PR-RR-04-RF1	Tripwire	$\frac{2}{9}$ (whole = $2+7 = 9$ parts).
PR-RR-04-CF1	Confirmation	$\frac{5}{9}$ (whole = $4+5 = 9$ parts).
PR-RR-04-CF1	Road Test	Windows are $\frac{5}{8}$ of the windows-and-doors total (whole = $5+3=8$).
PR-RR-05-RF1	Route Finder	2:3.
PR-RR-05-RF1	Tripwire	3:5.
PR-RR-05-CF1	Confirmation	4:6 (or simplified 2:3).
PR-RR-05-CF1	Road Test	9:7.
PR-RR-06-RF1	Route Finder	3:4 (GCF of 18 and 24 is 6).
PR-RR-06-RF1	Tripwire	2:3 (GCF of 16 and 24 is 8, not 2).
PR-RR-06-CF1	Confirmation	2:3 (GCF of 28 and 42 is 14).
PR-RR-06-CF1	Road Test	2:3 (GCF is 18).
PR-RR-07-RF1	Route Finder	Yes -- both parts scaled by 3.

Form ID	Item	Correct response
PR-RR-07-RF1	Tripwire	No -- 4:7 scaled by any single factor does not give 7:10 ($4 \times k = 7$ gives $k = 1.75$, but $7 \times 1.75 = 12.25 \neq 10$). Not equivalent.
PR-RR-07-CF1	Confirmation	Yes -- both scaled by 4.
PR-RR-07-CF1	Road Test	Not the same shade -- 3:8 scaled would need 9:24, not 9:14; the suggested mix is not equivalent.
PR-RR-08-RF1	Route Finder	Yes -- both parts $\times 3$.
PR-RR-08-RF1	Tripwire	No -- $8 \times 3 = 24 \neq 20$, so the ratios are not equivalent.
PR-RR-08-CF1	Confirmation	No -- $6 \times 3 = 18$ matches, but $11 \times 3 = 33 \neq 34$.
PR-RR-08-CF1	Road Test	Something is off -- $4 \times 3 = 12$ matches, but $9 \times 3 = 27 \neq 26$.
PR-RR-09-RF1	Route Finder	Yes -- both simplify to 2:3.
PR-RR-09-RF1	Tripwire	Matching simplest forms means both ratios describe the same underlying relationship once all common factors are removed -- that shared reduced form is the equivalence test itself.
PR-RR-09-CF1	Confirmation	Yes -- both simplify to 2:3, confirming the same underlying relationship.
PR-RR-09-CF1	Road Test	Yes, the same ratio -- both simplify to 2:3.
PR-RR-10-RF1	Route Finder	60 km/h ($180 \div 3$).
PR-RR-10-RF1	Tripwire	0.25 cups per serving ($5 \div 20$).
PR-RR-10-CF1	Confirmation	25 pages per minute.
PR-RR-10-CF1	Road Test	15 liters per minute.
PR-RR-11-RF1	Route Finder	Store B (\$0.70/pen vs. Store A's \$0.80/pen).
PR-RR-11-RF1	Tripwire	Store B has the lower unit price (\$0.45/apple vs. Store A's \$0.50/apple).
PR-RR-11-CF1	Confirmation	Brand X (\$1.25/battery vs. Brand Y's \$1.35/battery).
PR-RR-11-CF1	Road Test	The 5-pack (\$1.45/bar vs. the 3-pack's \$1.50/bar).
PR-RR-12-RF1	Route Finder	5 cups (unit rate 0.25 cups/serving $\times 20$).
PR-RR-12-RF1	Tripwire	30 liters total after 6 minutes (rate is 5 L/min $\times 6$ minutes), not $15 + 3$.

Form ID	Item	Correct response
PR-RR-12-CF1	Confirmation	90 boxes (rate 6 boxes/min $\times$ 15).
PR-RR-12-CF1	Road Test	100 units (5 units/hour $\times$ 20).
PR-RR-13-RF1	Route Finder	Brand A: \$0.008/g. Brand B ($\$ \frac{9}{1200}$ g = \$0.0075/g) is slightly better.
PR-RR-13-RF1	Tripwire	Depends on the actual volume per dollar in a common unit -- “more items” alone does not establish better value without converting to a shared unit and computing the rate.
PR-RR-13-CF1	Confirmation	Cereal A: \$0.0094/g. Cereal B: \$0.0096/g -- Cereal A is slightly lower.
PR-RR-13-CF1	Road Test	750 mL bottle: \$0.0028/mL. 1.5 L (1500 mL) bottle: \$0.0026/mL -- the larger bottle is slightly better once converted to the same unit.
PR-RR-14-RF1	Route Finder	\$5/notebook vs. \$4/notebook; the second option is the better deal.
PR-RR-14-RF1	Tripwire	A complete answer states both computed unit rates and then the conclusion they support.
PR-RR-14-CF1	Confirmation	\$1.50/marker vs. \$1.40/marker; the second option is the better deal.
PR-RR-14-CF1	Road Test	A complete answer requires both unit rates stated, not just a size-based guess.
PR-RR-15-RF1	Route Finder	$x = 15$ ($3 \times 20 = 4 \times x$, so $60 = 4x$, $x = 15$).
PR-RR-15-RF1	Tripwire	$x = 6$ ($2 \times 15 = 5 \times x$, so $30 = 5x$, $x = 6$).
PR-RR-15-CF1	Confirmation	$x = 28$ ($7 \times 36 = 9 \times x$, $252 = 9x$, $x = 28$).
PR-RR-15-CF1	Road Test	$x = 25$.
PR-RR-16-RF1	Route Finder	$x = 6$ both ways: scale factor 1.5 ($4 \times 1.5 = 6$); cross-products $4 \times 9 = 6 \times x$, $36 = 6x$, $x = 6$.
PR-RR-16-RF1	Tripwire	Correct final answer should be determined by finding and fixing the error in one of the two methods, not by picking one arbitrarily.

Form ID	Item	Correct response
PR-RR-16-CF1	Confirmation	x=12 both ways: scale factor $\frac{4}{3}$ ($9 \times \frac{4}{3} = 12$); cross-products $9 \times 16 = 12 \times x$, $144 = 12x$, $x = 12$.
PR-RR-16-CF1	Road Test	The mismatch itself signals an error in one method; it must be found and fixed, not resolved by picking either number.
PR-RR-17-RF1	Route Finder	No -- at 0 items the cost is \$5, not \$0 (fails the zero-input test).
PR-RR-17-RF1	Tripwire	The specific evidence must be named: e.g., the value at zero input is nonzero, or the rate changes across the table.
PR-RR-17-CF1	Confirmation	No -- the value is nonzero at 0 uses (\$12), so it fails the zero-input test even though the rate of change is technically 0, not varying.
PR-RR-17-CF1	Road Test	No -- cost at 0 usage is still the flat fee, not \$0, so it's not proportional to usage.
PR-RR-18-RF1	Route Finder	No -- it does not pass through (0,0), regardless of the constant rate of increase.
PR-RR-18-RF1	Tripwire	No -- the difference is constant (4) but the quotient is not (6, 5, 4.67...), and it does not pass through (0,0) given the pattern (value at 0 would be 2, not 0).
PR-RR-18-CF1	Confirmation	No -- although it passes through the origin, the quotient is not constant ($\frac{5}{5} = 1$, $\frac{8}{10} = 0.8$).
PR-RR-18-CF1	Road Test	Proportional -- constant quotient of 20, and \$0 at 0 minutes is consistent.
PR-RR-19-RF1	Route Finder	k=5 in both -- the table's quotient ($\frac{10}{2} = 5$) matches the equation's coefficient.

Form ID	Item	Correct response
PR-RR-19-RF1	Tripwire	The same constant k must be identifiable in the table's quotient, the graph's steepness, and the equation's coefficient.
PR-RR-19-CF1	Confirmation	Yes -- $k=9$ in both ($\frac{36}{4}=9$, and the graph point gives $\frac{9}{1}=9$).
PR-RR-19-CF1	Road Test	They match only if the same constant (the cost per unit) appears in all three; that shared value is what must be checked.
PR-RR-20-RF1	Route Finder	Any correct pair, e.g. $y=4x$ (proportional, passes through origin) and $y=4x+1$ (not proportional, does not).
PR-RR-20-RF1	Tripwire	The non-proportional table must be checked and revised so it genuinely fails the origin or quotient test.
PR-RR-20-CF1	Confirmation	Any correctly verified contrasting pair in a new context (e.g., time vs. temperature).
PR-RR-20-CF1	Road Test	Pay-as-you-go (no base fee) is proportional; subscription (with a base fee) is not, since it doesn't pass through (0,0).
PR-RR-21-RF1	Route Finder	40 km (5×8).
PR-RR-21-RF1	Tripwire	6.5 cm ($65 \div 10$).
PR-RR-21-CF1	Confirmation	21 feet (3.5×6).
PR-RR-21-CF1	Road Test	20 cm ($1740 \text{ cm} \div 87$).
PR-RR-22-RF1	Route Finder	Yes -- both pairs scale by 1.5 ($4 \rightarrow 6, 6 \rightarrow 9$).
PR-RR-22-RF1	Tripwire	Not similar -- $5 \rightarrow 9$ is a factor of 1.8, not 2, so not all corresponding pairs share the same factor.
PR-RR-22-CF1	Confirmation	Not similar -- $6 \rightarrow 9$ is factor 1.5, but $10 \rightarrow 14$ is factor 1.4.
PR-RR-22-CF1	Road Test	Similar -- both dimensions scale by exactly 1.5.
PR-RR-23-RF1	Route Finder	15 cm (5×3).
PR-RR-23-RF1	Tripwire	Each length must be scaled by 4 individually (24 cm and 16 cm), not combined first.

Form ID	Item	Correct response
PR-RR-23-CF1	Confirmation	28 m and 16 m.
PR-RR-23-CF1	Road Test	0.2 m and 0.125 m (each divided by 20).
PR-RR-24-RF1	Route Finder	Percent (specifically, percent decrease).
PR-RR-24-RF1	Tripwire	This is a ratio problem (part-to-part-to-whole), requiring the total-parts method, not a percent calculation.
PR-RR-24-CF1	Confirmation	Scale (map/drawing to real conversion).
PR-RR-24-CF1	Road Test	Rate/proportionality check -- not proportional (nonzero cost at 0 texts).
PR-RR-25-RF1	Route Finder	12 boys ($\frac{3}{7}$ of 28).
PR-RR-25-RF1	Tripwire	Correct setup requires matching which quantity is the numerator on both sides, consistent with the classified anchor model's own method (PR-RR-15).
PR-RR-25-CF1	Confirmation	\$45; verify by substituting back ($60 \times 0.75 = 45$).
PR-RR-25-CF1	Road Test	\$8 (unit rate $\$ \frac{2}{3}$ per apple $\times$ 12, or scale factor 4 applied to \$2).

Chapter 1 -- What Proportional Reasoning Means

Chapter Support

Materials: Use the chapter diagrams, a ruler or measuring strip, sticky notes or counters, a coordinate grid, and two sets of cards labeled with input/output pairs. If available, provide a calculator only after students have estimated and identified the relationship.

Differentiation

Support for students who are not yet secure

Begin with unit strips and physical grouping before using equations. Keep the input and output quantities in separate labeled rows. Use only whole-number rates at first, then move to decimals. Ask the student to complete one row at a time and say the units aloud: “\$3 for 1 notebook,” “\$6 for 2 notebooks.” Return to the zero case often.

For students with language needs, provide sentence frames:

“The input represents _____.”

“The output represents _____.”

“The unit rate is _____ per _____.”

“The relationship is proportional because _____.”

“The relationship is not proportional because _____.”

Extension for students ready to generalize

Ask students to compare two methods for the same problem, create a proportional and non-proportional pair with the same additive pattern, interpret fractional constants of proportionality, and explain why (0,0) and (1,k) are especially informative points on a proportional graph.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. A taxi charges a flat fee plus a per-km rate. Given only two (distance, cost) pairs, determine whether the relationship is proportional, and if not, find the flat fee.
2. Two students each claim a different quantity is 'the constant of proportionality' for the same table. Determine which student is correct and explain the other's error.
3. Build a real-world context where the constant of proportionality is a fraction less than 1, and explain what that constant means in words.

4. Given a graph with two labeled points, neither of which is the origin, determine whether the relationship could be proportional, and state what additional information would confirm it.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-01 Decide whether a relationship is proportional

Mathematical claim: Proportional only if $y \div x$ is constant for every nonzero pair; checking (0,0) is a fast robustness safeguard, not a second independent condition.

Route Finder

Item: A table shows (1,4), (2,8), (3,12). Is this relationship proportional? Show the check you used.

Correct response: Yes. $4 \div 1 = 4$, $8 \div 2 = 4$, $12 \div 3 = 4$ -- constant quotient, and (0,0) fits.

Tripwire

Observed: A shop charges a \$3 setup fee, then \$2 per item. At 2 items the cost is \$7; at 4 items it is \$11. Is this relationship proportional?

Likely wrong response: "Yes, proportional -- it goes up by \$2 every time."

Correct response: No. The difference is constant (\$2 per item) but the quotient is not ($7 \div 2 = 3.5$, $11 \div 4 = 2.75$), and the cost at 0 items is \$3, not \$0.

Likely reasoning: *may indicate a constant difference is being mistaken for a constant quotient (additive change read as multiplicative)*

Confirm (approach): Give a second, independently-different item: a table with a different constant difference and a different nonzero starting value. If the same additive read recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A gym charges no fee and \$5 per visit; another gym charges a \$10 membership plus \$5 per visit. Both show up as \$5/visit on a receipt. Which gym's total cost is proportional to number of visits?

Correct response: The no-fee gym only. Its cost at 0 visits is \$0 and the quotient is constant at 5; the membership gym's cost at 0 visits is \$10, not \$0.

Teach: Double number line + table pair, side by side. Ask the Talk Prompt directly: “What stays the same, the difference or the quotient?” Require $y \div x$ computed for two nonzero pairs before any yes/no answer.

Fresh Check (Recalculation Check)

Item: A relationship costs \$5 at 1 item and \$8 at 2 items. Proportional or not?

Correct response: Not proportional. Quotients are 5 and 4 -- not constant.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Without being told which chapter this is from: a delivery service's price list shows 2 packages for \$9 and 5 packages for \$22.50. Is pricing proportional to package count?

Correct response: Yes -- $9 \div 2 = 4.50$ and $22.50 \div 5 = 4.50$, constant quotient, and \$0 for 0 packages is consistent with the rate.

Route: If check passes: move to PR-RR-02. If check fails again: reteach with concrete counters before returning to the double number line.

PR-RR-02 Find and apply a unit rate

Mathematical claim: A unit rate is found by isolating “for one,” then scaled; scaling by addition instead of multiplication breaks the relationship.

Route Finder

Item: 4 notebooks cost \$12. What is the cost of 7 notebooks?

Correct response: \$21 (unit rate \$3/notebook, then $7 \times \$3$).

Tripwire

Observed: 6 pencils cost \$9. A student is asked for the cost of 10 pencils, and answers \$13 by adding \$4 (the jump from 6 to 10) onto \$9.

Likely wrong response: \$13, from adding the difference in pencils onto the difference in cost.

Correct response: \$15 (unit rate \$1.50/pencil, $10 \times \$1.50$).

Likely reasoning: may indicate a difference is being applied where a scale factor is required (additive scaling instead of multiplicative)

Confirm (approach): Give a second scaling item using a different context and different numbers. If the additive shortcut recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A runner covers 3 laps in 12 minutes at a constant pace. Explain, in words, how you would find the time for 7 laps -- don't just compute it.

Correct response: Find the time for 1 lap first ($12 \div 3 = 4$ minutes), then multiply by 7 (28 minutes) -- describes the unit-first method, not an additive shortcut.

Teach: Walk the double number line one step at a time: find the value for ONE unit first, in isolation, before scaling to the target amount.

Fresh Check (Recalculation Check)

Item: 3 tickets cost \$21. What do 8 tickets cost?

Correct response: \$56 (unit rate \$7/ticket).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe blog says 5 servings need 750 mL of broth. A reader wants to know the broth for 9 servings, without being told this is a unit-rate problem.

Correct response: 1350 mL ($150 \text{ mL/serving} \times 9$).

Route: If check passes: move to PR-RR-03. If check fails: reteach the one-unit-first step explicitly before any scaling.

PR-RR-03 Justify a proportionality claim across representations

Mathematical claim: The same constant must be identifiable in a table, graph, equation, and word context, with the connection stated, not just the number.

Route Finder

Item: A relationship is $y = 6x$. State what 6 means in a sentence using units.

Correct response: Any correct unit-rate sentence, e.g. "6 is the cost in dollars for every 1 item."

Tripwire

Observed: A student correctly computes that a relationship's constant is 4.5, then is asked to explain what 4.5 means, and states only "4.5 is the answer."

Likely wrong response: No explanatory sentence given, only the number restated.

Correct response: A stated sentence such as "4.5 means \$4.50 per item" or "4.5 hours per trip," with units.

Likely reasoning: *may indicate correct computation without a stated connecting explanation, an explanation gap rather than a computation gap*

Confirm (approach): Give a second open-response item requiring a connecting sentence in a different context. If the sentence is skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Two runners' speeds are shown as points on separate graphs, each passing through the origin. One graph shows the point (2,14); state what the slope means for that runner, with units.

Correct response: Any correct sentence, e.g. "7 km per hour -- the runner covers 7 km every hour."

Teach: Require the sentence explicitly: "[rate] means ___ per ___." Do not accept the number alone as a complete answer.

Fresh Check (Recalculation Check)

Item: A relationship is $y = 12x$. Explain what 12 means, with units, for a context of your choice.

Correct response: Any correct sentence naming 12 as a per-unit rate with stated units.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A water tank's fill rate is described only as "9 liters every 3 minutes," with no chapter context given. Someone asks what this tells you about 1 minute. Answer in a full sentence.

Correct response: 3 liters per minute -- the tank fills 3 liters every minute, found by dividing 9 by 3.

Route: If check passes: chapter secure, move to PR-FR-01 (Chapter 2). If check fails: reteach requiring the sentence on every subsequent item this week.

Chapter 2 -- Fractions as Relationships

Chapter Support

Materials: Use paper strips, fraction circles or rectangles, counters or linking cubes, a 0–2 number line, colored pencils, and the four chapter diagrams. If physical manipulatives are unavailable, use folded paper strips and drawn groups. All essential meaning should remain available in black-and-white printing.

Differentiation

Students needing concrete support

Use physical strips or counters before symbols. Keep the whole visible and label it explicitly. Ask students to build the denominator first, then shade or select the numerator. For fraction-of-a-number problems, make equal groups with counters and say the unit fraction aloud: “One fifth of 30 is 6.” Then select the numerator number of groups.

Students needing language support

Use sentence frames:

“The whole is _____.”

“The denominator tells me _____.”

“The numerator tells me _____.”

“ab means _____ equal parts out of _____.”

“ab is greater than 1 because _____.”

“The answer is reasonable because the fraction is _____ than 1.”

Students ready to extend

Ask students to construct two contexts for the same fraction, compare equivalent fractions on a number line, derive a mixed number from an improper fraction, and explain why ab of n can be represented as $(a \times n) \div b$ when the quantities are compatible.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Two classes report the same fraction of students at an event. Given each class's total size, determine whether the actual number of students is the same, and explain why the fraction alone doesn't answer this.
2. Find a fraction of a quantity two ways: by computing the unit fraction first, and by an incorrect shortcut that stops early. Show both and explain what the shortcut is missing.
3. Compare two fractions with different denominators using a number line, then verify the same comparison using a common denominator, and confirm they agree.
4. Create a fraction greater than 1 and correctly place it on a number line, then explain why it can't sit between 0 and 1.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-FR-01 Understand a fraction as parts of a specified whole

Mathematical claim: A fraction only has meaning relative to a stated whole; equal fractions of different wholes are not equal amounts.

Route Finder

Item: Class A has $\frac{3}{4}$ of 20 students at an assembly. Class B has $\frac{3}{4}$ of 28 students at an assembly. Do the two classes have the same number of students there?

Correct response: No -- 15 students vs. 21 students. Same fraction, different wholes.

Tripwire

Observed: Two classes each report that $\frac{3}{4}$ of their students are at an activity. A student concludes the two classes must have the same number of students there.

Likely wrong response: "Yes, same fraction means same number."

Correct response: No, not necessarily -- the fraction is the same but the wholes (total class sizes) may differ.

Likely reasoning: *may indicate the whole is going untracked, so equal-looking fractions are treated as equal amounts*

Confirm (approach): Give a second item with two different unstated whole sizes attached to the same fraction. If the same conflation recurs, treat as confirmed.

Additional diagnostic note: If a student confuses denominator and answer, return to "one denominator-sized group" and ask for the size of one part.

Additional diagnostic note: If a student accepts unequal parts, use two strips with the same number of pieces and compare the sizes of the pieces.

Additional diagnostic note: If a student rejects fractions greater than 1, place values on a number line and rename them as mixed numbers.

Confirmation Item

Same claim, different surface form.

Item: A survey says $\frac{2}{3}$ of a school's teachers and $\frac{2}{3}$ of a school's students prefer online forms. The school has 40 teachers and 600 students. Is the number of people preferring online forms the same for both groups?

Correct response: No -- about 27 teachers vs. 400 students. Same fraction, very different wholes.

Teach: Fraction strip + number line together. Ask the Talk Prompt: “What is the whole, and what has been divided into equal parts?” Require the whole to be named before any comparison.

Fresh Check (Recalculation Check)

Item: $\frac{2}{5}$ of Team X's 15 players and $\frac{2}{5}$ of Team Y's 25 players are at practice. Same number at practice?

Correct response: No -- 6 players vs. 10 players.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Without naming a whole: “ $\frac{1}{2}$ of the trees in Park A and $\frac{1}{2}$ of the trees in Park B are oaks.” Is the number of oak trees necessarily the same in both parks?

Correct response: No -- the number of oak trees depends on each park's total number of trees, which may differ.

Route: If check passes: move to PR-FR-02. If check fails: reteach with concrete counters showing two different-sized wholes side by side.

PR-FR-02 Find a fraction of a quantity

Mathematical claim: Finding a/b of a quantity requires finding $1/b$ first, then scaling by a ; stopping after the unit fraction is incomplete, not a different correct answer. Highest-volume confusion in this chapter.

Route Finder

Item: Find $\frac{3}{5}$ of 20.

Correct response: 12 ($\frac{1}{5}$ of 20 = 4, then $3 \times 4 = 12$).

Tripwire

Observed: A student is asked to find $\frac{5}{6}$ of 24, and answers 4.

Likely wrong response: 4 -- the unit fraction ($\frac{1}{6}$) found but never scaled by the numerator.

Correct response: 20 ($\frac{1}{6}$ of 24 = 4, then $5 \times 4 = 20$).

Likely reasoning: may indicate the unit-fraction-only error: the scaling-by-numerator step is being skipped, the chapter's dominant error

Confirm (approach): Give a second fraction-of-quantity item with a different denominator and numerator. If the answer again matches the unit fraction alone, treat as confirmed.

Additional diagnostic note: If a student answers with the numerator only, the student needs an operator model with a clearly labeled whole.

Confirmation Item

Same claim, different surface form.

Item: A charity keeps $\frac{2}{9}$ of every donation for overhead. If a donation is \$63, how much goes to overhead? Show your steps as a story, not just numbers.

Correct response: \$14 -- find $\frac{1}{9}$ of \$63 first (\$7), then multiply by 2 (\$14).

Teach: Strip-folding: find the value of one part first as an explicit, visible step, then multiply by the number of parts needed as a separate, visible second step.

Fresh Check (Recalculation Check)

Item: Find $\frac{4}{7}$ of 35.

Correct response: 20 ($\frac{1}{7}$ of 35 = 5, then $4 \times 5 = 20$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A trail is $\frac{5}{8}$ finished after today's work. If the trail is 48 km total, how much is finished, presented with no mention of 'fraction of a quantity'?

Correct response: 30 km ($\frac{1}{8}$ of 48 = 6, then $5 \times 6 = 30$).

Route: If check passes: move to PR-FR-03. If check fails: reteach requiring both steps shown separately on every item, no exceptions, until it holds twice.

PR-FR-03 Compare fractions and justify using a common reference

Mathematical claim: Two fractions can only be compared directly once expressed against the same whole or reference (common denominator, benchmark, or number line).

Route Finder

Item: Which is greater, $\frac{3}{8}$ or $\frac{5}{12}$? Show the common reference you used.

Correct response: $\frac{5}{12}$ is greater (common denominator 24: $\frac{9}{24}$ vs. $\frac{10}{24}$).

Tripwire

Observed: A student compares $\frac{2}{5}$ and $\frac{3}{8}$ by saying “3 is bigger than 2, and 8 is bigger than 5, so $\frac{3}{8}$ is bigger” without converting to a common denominator.

Likely wrong response: Comparing numerators and denominators visually without a common reference.

Correct response: $\frac{2}{5}$ is actually greater (common denominator 40: $\frac{16}{40}$ vs. $\frac{15}{40}$) -- the digit-comparison shortcut gives the wrong answer here, which is exactly why the conversion step is required every time, not just when convenient.

Likely reasoning: may indicate numerators or denominators are being compared visually rather than converted to a shared reference first

Confirm (approach): Give a second comparison pair with unlike denominators where visual comparison would mislead. If the visual shortcut recurs, treat as confirmed.

Additional diagnostic note: If a student can calculate but cannot explain, require a model and sentence before further symbolic practice.

Confirmation Item

Same claim, different surface form.

Item: A juice recipe uses either $\frac{5}{9}$ cup or $\frac{7}{12}$ cup of concentrate, depending on the batch size chart. Which uses more concentrate per batch?

Correct response: $\frac{7}{12}$ cup (common denominator 36: $\frac{20}{36}$ vs. $\frac{21}{36}$).

Teach: Require a stated common reference (common denominator, benchmark, or number line placement) before accepting any comparison as justified.

Fresh Check (Recalculation Check)

Item: Which is greater, $\frac{5}{6}$ or $\frac{7}{9}$? State your common reference.

Correct response: $\frac{5}{6}$ is greater (common denominator 18: $\frac{15}{18}$ vs. $\frac{14}{18}$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two containers are marked $\frac{4}{7}$ full and $\frac{3}{5}$ full. Without mentioning fractions or comparisons directly, ask which container has more room left.

Correct response: The $\frac{4}{7}$ container has more room left (it's less full: $\frac{4}{7} = \frac{20}{35}$, $\frac{3}{5} = \frac{21}{35}$, so $\frac{3}{5}$ is fuller, leaving less room).

Route: If check passes: chapter secure, move to PR-FR-04 (Chapter 3). If check fails: reteach the common-denominator step as mandatory before any comparison claim.

Chapter 3 -- Fraction Operations for Scaling and Sharing

Chapter Support

Materials: Prepare fraction strips marked in halves, thirds, fourths, sixths, eighths, and tenths; paper strips; fraction circles; counters; measuring cups; small containers; a 0–2 number line; and the chapter diagrams. Students should be able to physically join, separate, scale, share, and group quantities.

Differentiation

Students needing concrete support

Use small whole-number quantities and physical materials. For addition and subtraction, use one strip per fraction and physically rename the units. For multiplication, make equal groups with counters. For division, alternate between sharing counters among people and counting how many fraction-sized groups fit.

Students needing language support

Use the frames:

“The action is _____, so I will _____.”

“The denominator names _____-sized parts.”

“The numerator tells me _____.”

“I know the answer is reasonable because _____.”

“This is sharing division because _____.”

“This is grouping division because _____.”

Students ready to extend

Ask students to solve a problem using two representations, compare multiplication by a fraction less than 1 with multiplication by a fraction greater than 1, create contexts for the same equation, and explain why the reciprocal appears in the symbolic division algorithm after the grouping meaning is established.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. A recipe scales by a fraction less than 1 for a smaller batch, then a different fraction for sharing among people. Find the final amount using both operations, and identify which was scaling and which was sharing.
2. Two fraction division problems use the same numbers but ask different questions (how many groups fit vs. what size is one group). Solve both and explain why the answers differ.
3. Create a subtraction problem where the two fractions already share a denominator, and a second where they don't. Explain why one requires an extra step.
4. A quantity is multiplied by a fraction greater than 1 in one problem and a fraction less than 1 in another. Predict which result is larger before calculating, then verify.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-FR-04 Multiply a fraction to scale a quantity

Mathematical claim: Multiplying by a fraction scales a quantity; the operation is chosen by recognizing the scaling action, not by keyword-matching or defaulting to multiplication.

Prerequisites: PR-FR-02

Route Finder

Item: A recipe for 1 batch uses $\frac{2}{3}$ cup of oats. How much oats for 5 batches?

Correct response: $\frac{10}{3}$ cups, or 3 and $\frac{1}{3}$ cups ($\frac{2}{3} \times 5$).

Tripwire

Observed: A student is asked to find $\frac{3}{4}$ of a $\frac{2}{3}$ -cup measure, and multiplies straight across to get $\frac{6}{12}$, but does not simplify or check the size makes sense relative to the original $\frac{2}{3}$.

Likely wrong response: Leaves the answer as $\frac{6}{12}$ unsimplified, or does not notice/check that the result should be smaller than $\frac{2}{3}$.

Correct response: $\frac{1}{2}$ cup ($\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$), smaller than the original $\frac{2}{3}$ cup, which makes sense since a fraction less than 1 was applied.

Likely reasoning: may indicate the result of multiplying by a fraction less than 1 is not being checked against the expectation that it should be smaller

Confirm (approach): Give a second multiplication-of-fractions item and ask the student to state whether the result should be larger or smaller before computing. If the size-check is skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A wall takes $\frac{5}{6}$ of a can of paint. A smaller wall needs $\frac{2}{3}$ of that amount. How much paint does the smaller wall need?

Correct response: $\frac{5}{9}$ of a can ($\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$).

Teach: Common-unit strip + labeled group diagrams. Talk Prompt: “What is happening to the quantity?” Require a before-computing prediction of larger/smaller.

Fresh Check (Recalculation Check)

Item: Find $\frac{2}{5}$ of $\frac{3}{4}$ of a cup. Is your answer larger or smaller than $\frac{3}{4}$ cup, and why?

Correct response: $\frac{3}{10}$ cup; smaller, because a fraction less than 1 was applied.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A garden plot is $\frac{3}{4}$ of an acre. A farmer plants vegetables on $\frac{4}{5}$ of the plot, described with no scaling language at all.

Correct response: $\frac{3}{5}$ of an acre ($\frac{4}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5}$).

Route: If check passes: move to PR-FR-05. If check fails: reteach with physical strip-folding before returning to symbolic multiplication.

PR-FR-05 Add and subtract fractions with unlike denominators

Mathematical claim: Fractions must share a common denominator before numerators combine; adding numerators and denominators straight across produces a value with no consistent meaning.

Prerequisites: PR-FR-01

Route Finder

Item: $\frac{1}{3} + \frac{1}{4} = ?$

Correct response: $\frac{7}{12}$ (common denominator 12: $\frac{4}{12} + \frac{3}{12}$).

Tripwire

Observed: A student adds $\frac{2}{5} + \frac{1}{3}$ and answers $\frac{3}{8}$ by adding numerators and denominators straight across.

Likely wrong response: $\frac{3}{8}$, from adding numerators and denominators independently.

Correct response: $\frac{11}{15}$ (common denominator 15: $\frac{6}{15} + \frac{5}{15}$).

Likely reasoning: may indicate numerators and denominators are being combined straight across without converting to a common denominator first

Confirm (approach): Give a second unlike-denominator item, and a third item where denominators already match, to check for both the missing conversion and any habitual over-application. If the straight-across error recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A hiker walks $\frac{2}{3}$ of a trail on day one and $\frac{1}{4}$ more of the trail on day two. What fraction has the hiker covered after two days?

Correct response: $\frac{11}{12}$ (common denominator 12: $\frac{8}{12} + \frac{3}{12}$).

Teach: Rename to a common unit before combining, shown as an explicit, separate, visible step every time.

Fresh Check (Recalculation Check)

Item: $\frac{3}{8} + \frac{1}{6} = ?$

Correct response: $\frac{13}{24}$ (common denominator 24: $\frac{9}{24} + \frac{4}{24}$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two paint cans hold $\frac{3}{10}$ and $\frac{1}{4}$ of a gallon. Combined, without mentioning 'add,' how much paint is that?

Correct response: $\frac{11}{20}$ of a gallon (common denominator 20: $\frac{6}{20} + \frac{5}{20}$).

Route: If check passes: move to PR-FR-06. If check fails: reteach the common-unit renaming step as a mandatory first line of work on every item.

PR-FR-06 Divide fractions and interpret what the quotient means

Mathematical claim: Dividing by a fraction less than 1 produces a larger result, the reverse of dividing by a whole number greater than 1; the quotient's meaning depends on which quantity divides which.

Prerequisites: PR-FR-04

Route Finder

Item: How many $\frac{3}{4}$ -cup servings fit in 6 cups?

Correct response: 8 servings ($6 \div \frac{3}{4} = 8$).

Tripwire

Observed: A student divides $4 \div \frac{1}{2}$ and answers 2, reasoning that dividing always makes a number smaller.

Likely wrong response: 2, from assuming division always shrinks the result.

Correct response: 8 ($4 \div \frac{1}{2} = 8$ -- how many halves fit into 4).

Likely reasoning: may indicate dividing by a fraction is being assumed to always shrink the result, by analogy with whole-number division

Confirm (approach): Give a second division-by-a-fraction item and ask the student to predict larger/smaller before computing. If the shrink assumption recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A ribbon is 9 meters long. Pieces of $\frac{3}{5}$ meter are cut from it. How many full pieces can be cut?

Correct response: 15 pieces ($9 \div \frac{3}{5} = 15$).

Teach: Common-unit strip showing directly how many fractional-sized groups fit inside the whole quantity, built concretely before any symbolic shortcut.

Fresh Check (Recalculation Check)

Item: How many $\frac{2}{3}$ -cup servings fit in 8 cups? Predict larger or smaller than 8 first.

Correct response: 12 servings; larger than 8, since fractional-sized groups are smaller than a whole cup.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A tank holds 10 liters. Bottles that hold $\frac{5}{6}$ liter each are being filled from it, phrased as a bottling-line question with no operation named.

Correct response: 12 bottles ($10 \div \frac{5}{6} = 12$).

Route: If check passes: chapter secure, move to PR-PC-01 (Chapter 4). If check fails: reteach with concrete fitting-in models before returning to the algorithm.

Chapter 4 -- Fractions, Decimals, and Percents as Three Languages

Chapter Support

Materials: Prepare fraction strips, 100-square grids, place-value charts, number lines from 0 to 2, cards labeled with equivalent forms, calculators for checking division, and the chapter diagrams. The card set should include equivalent and near-miss values such as 0.6, 6%, 60%, 35, 0.06, and 600%.

Differentiation

Students needing concrete support

Start with 12, 14, 34, and 1. Use strips and 100-square grids before less familiar fractions. Keep the same value visible in all three forms. Have students point to the same location before writing symbols.

Students needing language support

Use these sentence frames:

“The whole is _____.”

“The fraction means _____ equal parts.”

“The decimal has _____ tenths/hundredths/thousandths.”

“The percent means _____ out of 100.”

“The forms are equivalent because _____.”

“The most efficient route is _____ because _____.”

Students ready to extend

Include repeating decimals, fractions such as $\frac{7}{16}$, values greater than 1, multiple conversion routes, and conversion-error creation. Ask students to explain why a decimal terminates when the

simplified denominator has only factors of 2 and 5, while keeping the focus on representation and meaning.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given the same value expressed as a fraction, decimal, and percent with one form deliberately wrong, identify which one doesn't match and correct it.
2. Explain a real situation where a percent above 100% makes sense, and a different situation where it would not.
3. Convert a repeating decimal to a percent and explain how much precision is lost by rounding to the nearest whole percent.
4. Two students compare 0.6 and 60% and disagree about whether they're equal. Resolve the disagreement and explain the source of confusion.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-PC-01 Convert fluently among fraction, decimal, and percent forms

Mathematical claim: Fraction, decimal, and percent are three names for the same value; converting requires tracking place value (tenths vs. hundredths) and the direction of the $\times 100/\div 100$ step.

Route Finder

Item: Write 0.7 as a percent, and write 7% as a decimal. Are these the same value?

Correct response: $0.7 = 70\%$; $7\% = 0.07$. Not the same value -- this pair tests the tenths/hundredths distinction directly.

Tripwire

Observed: A student converts 0.6 to a percent and writes 6%.

Likely wrong response: 6%, from treating the decimal's digit directly as the percent number without the $\times 100$ step.

Correct response: 60% ($0.6 = \frac{60}{100}$).

Likely reasoning: may indicate tenths are being confused with hundredths, the chapter's dominant error

Confirm (approach): Give a second conversion pair using a different tenths value. If the same digit-copying pattern recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A spreadsheet cell shows 0.09. A student needs to enter the equivalent percent in a different cell. What should they type?

Correct response: 9%.

Teach: Shared number line and hundred grid, side by side. Talk Prompt: "Does this conversion change the value, or only the form?" Require the hundredths grid shaded, not just the shortcut applied.

Fresh Check (Recalculation Check)

Item: Write 0.4 as a percent, and 4% as a decimal.

Correct response: $0.4 = 40\%$; $4\% = 0.04$.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A weather app shows a 0.3 chance of rain, with no percent sign anywhere. A friend asks what percent chance that is.

Correct response: 30%.

Route: If check passes: move to PR-PC-02. If check fails: reteach on the hundred grid physically before returning to the shortcut.

PR-PC-02 Recognize and justify that a value can exceed 100%

Mathematical claim: Percent is not capped at 100; a percent greater than 100% describes more than the original whole, and this is a legitimate, common outcome, not an error.

Prerequisites: PR-PC-01

Route Finder

Item: A plant's height went from 20 cm to 45 cm. What percent of the original height is the new height?

Correct response: 225% ($\frac{45}{20} = 2.25 = 225\%$).

Tripwire

Observed: A quantity grows from 40 to 90. A student is asked what percent the new amount is of the original, and answers 50%.

Likely wrong response: 50%, from computing the percent increase (the change) instead of the percent the final amount represents of the original.

Correct response: 225% ($\frac{90}{40} = 2.25$).

Likely reasoning: may indicate the final amount is being confused with the change amount

Confirm (approach): Give a second growth item and require both the change amount and the final-amount percent stated separately. If the two are conflated again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A stock price goes from \$40 to \$54. What percent of the original price is the new price?

Correct response: 135% ($\frac{54}{40} = 1.35$).

Teach: Hundred grid extended past one full grid to show the >100% case visually; require both quantities (change and final) labeled separately.

Fresh Check (Recalculation Check)

Item: A quantity grows from 30 to 42. State both the percent increase and the percent the final amount represents of the original.

Correct response: 40% increase; 140% of the original.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A town's population went from 2,000 to 5,000 over ten years, described purely as a population report with no percent language.

Correct response: 250% of the original ($\frac{5000}{2000} = 2.5$).

Route: If check passes: move to PR-PC-03. If check fails: reteach with the extended grid until both quantities are reliably distinguished.

PR-PC-03 Justify which form fits a given purpose

Mathematical claim: No single form is universally correct; the best form depends on the task, and comparing across forms requires converting to a shared form first.

Prerequisites: PR-PC-01

Route Finder

Item: You want to compare a 25% discount and a $\frac{3}{10}$ discount. Which form would you convert to, and why?

Correct response: Any correctly justified common form (e.g., convert both to percent: 25% vs. 30%) with a stated reason.

Tripwire

Observed: A student is asked whether $\frac{3}{4}$ or 80% is the better way to describe a completion rate, and answers “80% is correct” without discussing purpose.

Likely wrong response: Declares one form the single correct answer with no purpose-based reasoning.

Correct response: Either can be correct depending on purpose; the answer must discuss why (e.g., percent for comparing across different totals, fraction for exact part-counting), not declare one form universally right.

Likely reasoning: *may indicate one form is being treated as universally correct rather than purpose-dependent*

Confirm (approach): Give a second purpose-comparison item in a different context. If a single form is again declared correct with no reasoning, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: For adjusting a recipe that serves 8 to serve 3, would you rather work with a fraction or a decimal multiplier? Justify.

Correct response: Any reasonably justified answer (fraction $\frac{3}{8}$ keeps exactness; decimal 0.375 is easier to multiply on a calculator).

Teach: Require the connecting sentence stating why a form fits the specific purpose, not just the converted number.

Fresh Check (Recalculation Check)

Item: For reporting exam scores to parents, would you use a fraction, decimal, or percent? Justify.

Correct response: Any reasonably justified choice (percent is conventional for scores and allows comparison across different total points).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A nutrition label and a sale sign both describe “25% of daily value” and “ $\frac{1}{4}$ off,” with no instruction to compare forms.

Correct response: Both describe the same proportion ($25\% = \frac{1}{4}$); which is clearer depends on the audience and purpose.

Route: If check passes: chapter secure, move to PR-PC-04 (Chapter 5). If check fails: reteach by contrasting two purposes side by side.

Chapter 5 -- Percent Means Out of 100

Chapter Support

Materials: Prepare 100-square grids, strips divided into 100 sections, counters, percent cards, fraction and decimal cards, number lines from 0% to 150%, double number-line templates, and the chapter diagrams. Use two grids when modeling values greater than 100%.

Differentiation

Students needing concrete support

Use 100-grid problems first. Restrict the first examples to 10%, 25%, 50%, 75%, and 100%. Then introduce non-benchmark values such as 35% and 62%. Keep the whole as a physical collection of counters or a labeled strip.

Students needing language support

Use the sentence frames:

“The whole is _____, so 100% means _____.”

“_____ % means _____ out of 100.”

“The answer should be less than/equal to/greater than the whole because _____.”

“The same percent gives a different amount because _____.”

“I used _____ as a benchmark.”

Students ready to extend

Ask students to decompose percentages such as 35% into 25% + 10%, compare multiple methods for percent-of problems, interpret 250% in several contexts, and explain when a percent above 100% is meaningful or impossible under a stated constraint.

Strategy Studio

Already published in the print book's Chapter 5 back matter; reproduced here for a complete pack.

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Show 35% as a hundred-grid description, a fraction, and a decimal.
2. A class has 18 students and 6 are absent. Is the absence rate 6%, 18%, or another percent? Identify the whole first.
3. A student says 125% is impossible because percent cannot exceed 100. Give a context in which 125% makes sense.
4. Write a percent problem whose answer is exactly 12.5%. Then write a different problem with the same answer but a different whole.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-PC-04 Convert a percent using the hundredths meaning

Mathematical claim: Percent literally means out of 100; conversions start from that hundredths meaning, not a memorized, disconnected shortcut.

Prerequisites: PR-PC-01

Route Finder

Item: 40% of a class of 25 students are absent. How many students is that?

Correct response: 10 students ($40\% = \frac{40}{100}$, $\frac{40}{100} \times 25 = 10$).

Tripwire

Observed: A student is asked to find 30% of a group of 40 people and treats 100% as literally 100 people, becoming confused about how to proceed.

Likely wrong response: Confusion or an incorrect setup from treating 100% as a literal count of 100 objects.

Correct response: 12 people ($30\% = 0.30$, $0.30 \times 40 = 12$; the whole is 40 people, not 100).

Likely reasoning: *may indicate 100% is being treated as a literal count of 100 things rather than “all of the whole, however large”*

Confirm (approach): Give a second percent-of-a-whole item where the whole is a number other than 100 or a multiple of 100. If the confusion recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A school has 150 students, and 60% are enrolled in music class. How many students is that?

Correct response: 90 students (0.60×150).

Teach: Percent bar + double number line. Talk Prompt: “35 out of what? What is 100% here?” Require the actual whole labeled on the bar before calculating.

Fresh Check (Recalculation Check)

Item: 20% of a group of 65 people are left-handed. How many people is that?

Correct response: 13 people.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A batch of 80 cookies is described as 15% burnt, in a bakery-inventory note with no mention of percent-of-a-whole.

Correct response: 12 cookies (0.15×80).

Route: If check passes: move to PR-PC-05. If check fails: reteach the percent bar with the actual whole labeled explicitly, not assumed to be 100.

PR-PC-05 Estimate a percent using benchmarks

Mathematical claim: Benchmarks (10%, 25%, 50%) are reference distances, not equally spaced labels; choosing the nearest one requires comparing actual distance.

Prerequisites: PR-PC-04

Route Finder

Item: Estimate 23% using the nearest benchmark. Which benchmark, and why?

Correct response: 25%, because 23 is closer to 25 than to 10 or 50 by actual distance.

Tripwire

Observed: A student is asked to estimate 48% using the nearest benchmark, and picks 10% because it is listed first on the benchmark chart.

Likely wrong response: Chooses a benchmark by position or familiarity rather than computed distance.

Correct response: 50%, because 48 is closer to 50 by actual distance.

Likely reasoning: *may indicate benchmark marks are being treated as equally spaced or chosen by familiarity rather than by measured distance*

Confirm (approach): Give a second estimation item with a value roughly equidistant from two benchmarks, forcing an actual distance comparison. If the same non-distance selection recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A poll shows 63% support for a proposal. Round to the nearest 25% benchmark and explain your choice.

Correct response: 75%, because 63 is 12 percentage points from 75% and 13 percentage points from 50%.

Teach: Percent bar with benchmark distances marked explicitly; require the distance to each benchmark stated before selecting the nearest one.

Fresh Check (Recalculation Check)

Item: Estimate 12% using the nearest benchmark, and state the distance to each benchmark you considered.

Correct response: 10%, distance 2, versus 25%, distance 13 -- 10% is nearer.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A jar is estimated to be about 77% full, in a context with no mention of percent estimation strategy at all.

Correct response: Nearest 25%-benchmark is 75% (distance 2), closer than 100% (distance 23).

Route: If check passes: move to PR-PC-06. If check fails: reteach with the benchmark distances drawn and measured explicitly on the bar.

PR-PC-06 Track the whole across a percent problem above 100%

Mathematical claim: Every percent value is meaningless without its whole; the final amount and the change amount are different quantities describing the same event.

Prerequisites: PR-PC-02

Route Finder

Item: A price increases by 20% from \$50. State both the change amount and the final amount.

Correct response: Change: \$10. Final: \$60 (120% of \$50).

Tripwire

Observed: A price increases by 15% from \$80, and a student states the final price is \$15 more without stating the actual final dollar amount, or confuses \$15 with the new total.

Likely wrong response: States only the change amount, or conflates the change amount with the final total.

Correct response: Change: \$12. Final: \$92.

Likely reasoning: *may indicate the whole is being described as important without being tracked concretely through to a distinguishing final answer*

Confirm (approach): Give a second percent-increase item and require both amounts labeled separately by name. If the two are conflated again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A charity's donations increased by 60% from \$500. State the change amount and the final amount.

Correct response: Change: \$300. Final: \$800.

Teach: Percent bar showing both the change segment and the final total segment as two separately labeled regions.

Fresh Check (Recalculation Check)

Item: A price increases by 35% from \$60. State the change amount and the final amount separately.

Correct response: Change: \$21. Final: \$81.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A company's headcount grew from 200 to 320 employees, framed as an HR memo with no percent terms used.

Correct response: Change: 120 employees (60% increase). Final: 320, which is 160% of the original 200.

Route: If check passes: chapter secure, move to PR-PC-07 (Chapter 6). If check fails: reteach with the two-segment bar until both amounts are reliably separated.

Chapter 6 -- Finding the Part, the Whole, or the Percent

Chapter Support

Materials: Prepare labeled part-percent-whole cards, percent bars, double number lines, 100-square grids, counters, sticky notes, and context cards. Each context card should require students to identify the whole before choosing a calculation.

Differentiation

Students needing concrete support

Use benchmark percentages first. For missing parts, build equal groups or use a percent strip. For missing wholes, build the 30% or 20% section first and enlarge it to 100%. For missing percents, compare a part strip with the whole strip and locate the relationship on a number line.

Students needing language support

Use these frames:

"The part is _____ because _____."

"The whole is _____ because it represents 100%."

"The percent is _____ because _____ out of the whole is selected."

"The missing quantity is _____, so I will _____."

"My check is valid because _____."

Students ready to extend

Ask students to solve the same problem using a benchmark, an equation, and a double number line. Include decimals and percents greater than 100% where the context permits. Ask students to reverse a problem: given the answer and one quantity, create a valid original question.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given a single set of three numbers, write three different word problems: one where the percent is missing, one where the whole is missing, and one where the part is missing.
2. A student always multiplies regardless of what's missing. Create a problem where this produces a wrong answer, and explain what should have been done instead.
3. Find the whole given a part and a percent above 100%, and explain why the whole is smaller than the given part in this case.
4. Verify a part/whole/percent answer using substitution rather than repeating the original calculation, and explain why this is a stronger check.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-PC-07 Identify which quantity is missing in a percent problem

Mathematical claim: Every percent problem states two of three quantities (part, whole, percent) and asks for the third; identifying which is missing must happen before calculating.

Prerequisites: PR-PC-04

Route Finder

Item: “15 is what percent of 60?” Which quantity is missing: the part, the whole, or the percent?

Correct response: The percent is missing (part=15 and whole=60 are both given).

Tripwire

Observed: A student reads “20 is 25% of what number?” and immediately multiplies 20 by 0.25 without first stating which quantity is missing.

Likely wrong response: 32 or another value from multiplying, having jumped to a calculation before identifying what's actually missing.

Correct response: The whole is missing (part=20, percent=25% are given); correct approach is $20 \div 0.25 = 80$.

Likely reasoning: may indicate the roles of part, whole, and percent are not being identified before a calculation is attempted

Confirm (approach): Give a second item with a different quantity missing (of the three) than the first. If role identification is skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: “45 students is 60% of the total enrolled.” Which quantity is missing?

Correct response: The whole is missing (part=45, percent=60% are given).

Teach: Labeled part-percent-whole bar. Talk Prompt: “Which of part, whole, and percent is missing?” Require the missing role named in words before any number is touched.

Fresh Check (Recalculation Check)

Item: “18 is what percent of 90?” Name which quantity is missing before solving.

Correct response: The percent is missing; answer is 20%.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe review says “9 reviewers out of the total gave 5 stars, which is 30% of everyone who reviewed,” with no percent-problem framing.

Correct response: The whole (total reviewers) is missing; part=9, percent=30%.

Route: If check passes: move to PR-PC-08. If check fails: reteach requiring the missing-role sentence on every item before calculation is allowed.

PR-PC-08 Choose the operation based on which quantity is missing

Mathematical claim: Finding the part requires multiplying by the percent; finding the whole or the percent requires dividing. Defaulting to multiplication regardless of what's missing is this chapter's dominant error.

Prerequisites: PR-PC-07

Route Finder

Item: 24 is 40% of what number? Solve, showing your operation choice.

Correct response: 60 ($24 \div 0.40 = 60$ -- dividing, since the whole is missing).

Tripwire

Observed: A student solves three problems in a row (find the part, find the whole, find the percent) by multiplying every time, having correctly solved the first “find the part” item.

Likely wrong response: Multiplies regardless of which quantity is actually missing.

Correct response: For “find the whole” or “find the percent,” division is required, not multiplication.

Likely reasoning: *may indicate multiplication is being applied by default regardless of which quantity is missing, the chapter's central error*

Confirm (approach): Give a mixed set: find-the-part, find-the-whole, find-the-percent, in that order, and check whether the operation changes appropriately. If multiplication persists on all three, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: 36 is 45% of what number? Solve.

Correct response: 80 ($36 \div 0.45$).

Teach: Same labeled bar, now used to decide the operation from which slot is empty -- multiply into the part, divide out of the whole or percent.

Fresh Check (Recalculation Check)

Item: Solve all three: (a) 30% of 50, (b) 15 is 30% of what?, (c) 15 is what percent of 50?

Correct response: (a) 15, (b) 50, (c) 30%.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A jar of 250 marbles has some fraction removed, leaving 175. What percent were removed, asked with no mention of the three percent-problem types?

Correct response: 30% removed ($\frac{75}{250} = 0.30$).

Route: If check passes: move to PR-PC-09. If check fails: reteach the bar-based operation decision explicitly on every item until the pattern breaks.

PR-PC-09 Verify a part-whole-percent solution by substituting back

Mathematical claim: A solution can be verified by substituting the found value back into the original relationship, a genuinely independent check unlike repeating the same method.

Prerequisites: PR-PC-08

Route Finder

Item: You found that 25% of 80 is 20. Verify this by substituting back.

Correct response: $\frac{20}{80} = 0.25 = 25\%$. Confirmed.

Tripwire

Observed: A student solves 18 is what percent of 72 (getting 25%), and “checks” by redoing $18 \div 72$ the same way, arriving at the same number again.

Likely wrong response: Repeats the same division as the “check,” which cannot catch an error made the first time.

Correct response: Verification should substitute back into the relationship (part = percent $\times$ whole: $0.25 \times 72 = 18$ confirms it), not repeat the identical calculation.

Likely reasoning: *may indicate checking is being understood as repeating the same method rather than substituting back into the original relationship*

Confirm (approach): Give a second item and explicitly ask for a check that is NOT the same operation as the solving step. If the same-method repeat recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A classmate claims 32 is 40% of 80 and checks by dividing 32 by 80 again, getting 0.4 again. Is this a valid check? Explain.

Correct response: Not fully independent -- repeating the same division can't catch an error made in that same division. A substitution-style check ($0.40 \times 80 = 32$) or a different method is needed.

Teach: Require substitution of the found value back into part = percent $\times$ whole as the explicit verification step.

Fresh Check (Recalculation Check)

Item: You found that 14 is 20% of 70. Verify by substituting back, not by repeating the original division.

Correct response: $0.20 \times 70 = 14$. Confirmed.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Someone calculates that 21 is 30% of 70 and wants to know if they're right, with no instruction on how to verify.

Correct response: Yes -- substituting back, $0.30 \times 70 = 21$, confirmed.

Route: If check passes: chapter secure, move to PR-PC-10 (Chapter 7). If check fails: reteach substitution-based checking explicitly as a distinct step from solving.

Chapter 7 -- Percent Increase, Decrease, Discount, Tax, Markup, and Tip

Chapter Support

Materials: Prepare price cards, play money, original–change–final strips, percent bars, double number lines, calculators for checking, and context cards for discount, tax, tip, markup, increase, and decrease. Label the original amount as 100% on every model.

Differentiation

Students needing concrete support

Use \$10, \$20, \$40, and \$100 examples first. Build the percent amount with counters or a 100-square grid. Keep the original, change, and final portions physically separate. Have students state “subtract” or “add” before calculating.

Students needing language support

Use the sentence frames:

“The original amount is _____.”

“The percent change is _____% of _____.”

“The change amount is _____.”

“The final amount is _____ because I added/subtracted _____.”

“The answer is reasonable because _____.”

Students ready to extend

Use successive changes, final-percent shortcuts, reverse problems, markup and discount comparisons, and contexts where the base changes at a second step. Ask students to prove why two 20% discounts are not one 40% discount.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. A price undergoes a discount and a tax in the same transaction. Determine the final price two ways: step-by-step and with a single combined multiplier, and confirm they agree.
2. Explain why a 20% markup followed by a 20% discount does not return to the original price, using a specific example.
3. Create a word problem where the percent change direction is easy to get backwards, and explain what makes it tricky.
4. A student treats a percent as a dollar amount directly. Construct a problem where this error is especially easy to make, and explain why.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-PC-10 Complete the two-step percent increase/decrease calculation

Mathematical claim: A percent increase or decrease requires two steps: find the percent amount, then add or subtract it; the percent number itself is never the dollar amount.

Prerequisites: PR-PC-08

Route Finder

Item: A \$40 item is discounted 25%. What is the sale price?

Correct response: \$30 (25% of \$40 = \$10; \$40 - \$10 = \$30).

Tripwire

Observed: A \$60 item is discounted 30%, and a student answers that the sale price is \$30, treating the percent number directly as a dollar reduction pattern.

Likely wrong response: \$30, from treating “30%” as though it directly meant “\$30 off” without computing the percent amount from the actual price.

Correct response: \$42 (30% of \$60 = \$18; \$60 - \$18 = \$42).

Likely reasoning: may indicate the percent number is being treated as a dollar amount directly, the chapter's central error

Confirm (approach): Give a second discount or markup item with a percent number that does not numerically resemble a plausible dollar amount. If the substitution recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A \$90 jacket is discounted 15%. A friend says the sale price is \$75 because “you just subtract 15.” Is this correct? Explain.

Correct response: No -- 15% of \$90 is \$13.50, not \$15; the sale price is \$76.50.

Teach: Original-change-final bar, all three values labeled explicitly, with the percent-amount calculation shown as its own visible step before subtracting or adding.

Fresh Check (Recalculation Check)

Item: A \$75 item is marked up 20%. What is the new price?

Correct response: \$90 (\$15 markup + \$75).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A phone bill of \$65 has a late fee of 8% added, mentioned in a household-budget note with no percent-problem framing.

Correct response: \$70.20 (\$65 + \$5.20).

Route: If check passes: move to PR-PC-11. If check fails: reteach with the three-part labeled bar until the percent-to-dollar step is reliably separated.

PR-PC-11 Match the direction of change to the correct operation

Mathematical claim: Discount and decrease subtract; markup, tax, and tip add. Direction is determined by context, not assumed.

Prerequisites: PR-PC-10

Route Finder

Item: A price is marked up 15% from \$200. Does the price increase or decrease, and what is the new price?

Correct response: Increases; \$230.

Tripwire

Observed: An item is marked up 10% from \$50, and a student subtracts, getting \$45.

Likely wrong response: \$45, from subtracting when the context (markup) requires adding.

Correct response: \$55 (markup adds; $\$50 + \$5$).

Likely reasoning: *may indicate the direction of change is not being matched to the correct operation for this specific context*

Confirm (approach): Give a discount item immediately after a markup item with similar numbers, checking whether direction is reasoned about or the prior operation is just repeated. If direction is mismatched again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A \$120 service fee is discounted 5% for early payment. Does the price go up or down, and what's the new price?

Correct response: Down; \$114.

Teach: Original-change-final bar with an explicit check: "Does the context add or remove an amount?" before choosing the operation.

Fresh Check (Recalculation Check)

Item: A restaurant bill of \$40 has an 18% tip added. What is the total?

Correct response: \$47.20.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A savings account balance grows by 4% interest in one year, in a bank-statement description with no direction language.

Correct response: Increases -- interest is always added to a balance.

Route: If check passes: move to PR-PC-12. If check fails: reteach by sorting contexts into an add-pile and a subtract-pile before any calculation.

PR-PC-12 Reconcile two solution methods for a percent change

Mathematical claim: A percent-change answer should be reproducible by more than one method; a mismatch between two methods signals an error that must be resolved, not ignored.

Prerequisites: PR-PC-11

Route Finder

Item: A \$50 item is discounted 20%. Solve two ways: (1) find the discount then subtract, (2) use a single multiplier. Do both methods agree?

Correct response: \$40 both ways: (1) \$10 off, $\$50 - \$10 = \$40$; (2) $\$50 \times 0.80 = \$40$.

Tripwire

Observed: A student solves a 15% discount on \$80 two ways and gets \$68 with one method and \$72 with the other, and submits \$68 without addressing the mismatch.

Likely wrong response: Submits one answer without noting or resolving that the two methods disagreed.

Correct response: Correct answer is \$68 ($\$80 \times 0.85$); the student should notice and resolve the discrepancy, not just pick one.

Likely reasoning: *may indicate a mismatch between two solution methods is going unnoticed or unreconciled*

Confirm (approach): Give a second two-method item with an intentional planted error in one method, and check whether the student catches the mismatch. If it goes unnoticed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A \$200 item is marked up 12%. Solve using both the add-the-markup method and the single-multiplier method ($\times 1.12$). Do they agree?

Correct response: \$224 both ways.

Teach: Require the connecting sentence naming what was added or subtracted and why, and require both methods' answers stated side by side for comparison.

Fresh Check (Recalculation Check)

Item: Solve a 12% increase on \$150 two ways, and state whether they agree.

Correct response: \$168 both ways; if a student's two methods disagree, they must find and fix the error before finishing.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two students compute a 10% tip on a \$45 bill using different methods and get \$49.50 and \$50. Which is correct, and why does the mismatch matter?

Correct response: \$49.50 is correct (45×1.10); the mismatch shows one method has an error that must be found before trusting either answer.

Route: If check passes: chapter secure, move to PR-PC-13 (Chapter 8). If check fails: reteach requiring both methods on every item until mismatches are reliably caught.

Chapter 8 -- Multi-Step Percent Problems

Chapter Support

Materials: Prepare original–change–final strips, base cards, price cards, percent cards, calculators for verification, year-by-year interest tables, and the chapter diagrams. Make students physically move the “base” card after each step.

Differentiation

Students needing concrete support

Use \$100 examples and physical strips. Have students record each step in a table with columns for base, percent, change, and new base. Use one operation per line and delay multiplier notation until the sequence is secure.

Students needing language support

Use the frames:

“At this step, the base is _____.”

“The change is _____% of _____.”

“The new base is _____.”

“I cannot add the percentages directly because _____.”

“The final answer is reasonable because _____.”

Students ready to extend

Introduce composed multipliers, reverse engineering, simple versus compound growth, and order effects. Ask students to compare the outcomes of two changes in different orders and explain why multiplication of the factors is commutative while the contextual base interpretation still matters.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. A price is discounted, then taxed, then a loyalty discount is applied to the new price. Track the base at each of the three steps.
2. Explain why simple interest over 3 years is not the same as three separate one-year compound calculations, using numbers.
3. Create a reverse-percent problem (given the final amount, find the original) and explain why the forward operation would give a wrong answer if used directly.
4. A student says two opposite percent changes of the same size cancel out. Construct a counterexample with real numbers.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-PC-13 Track the base across multiple percent-change steps

Mathematical claim: In a multi-step percent problem, the base for each step is the previous step's result, not the original amount. Forgetting to update the base is this chapter's dominant error.

Prerequisites: PR-PC-11

Route Finder

Item: A \$100 item is discounted 20%, then the discounted price is taxed 8%. What is the final price?

Correct response: \$86.40 ($\$100 \times 0.80 = \$80$, then $\$80 \times 1.08 = \$86.40$).

Tripwire

Observed: A \$100 item is discounted 20%, then taxed 8%, and a student computes 8% tax on the original \$100 instead of the \$80 sale price.

Likely wrong response: \$88 (from taxing the original \$100 instead of the discounted \$80).

Correct response: \$86.40, taxing the \$80 sale price, not the original \$100.

Likely reasoning: may indicate the base is not being updated after the first change, with the original amount reused for the next step

Confirm (approach): Give a second two-step item with discount-then-tax reversed to increase-then-increase, checking whether the base-update habit transfers. If the base is not updated again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A \$250 laptop is discounted 15%, then a 6% tax is added to the sale price. What is the final price?

Correct response: \$225.25 ($\$250 \times 0.85 = \$212.50$, then $\times 1.06 = \$225.25$).

Teach: Base-tracking strip, with the base explicitly relabeled at each step before the next calculation begins.

Fresh Check (Recalculation Check)

Item: A \$150 item is discounted 10%, then taxed 5%. Show the base used at each step.

Correct response: \$135 after discount (base for tax step), then \$141.75 final.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A store marks up wholesale cost by 40%, then offers a 10% loyalty discount on the marked-up price, framed as a pricing-strategy question with no 'base' language.

Correct response: Final price is 126% of wholesale cost ($1.40 \times 0.90 = 1.26$), so the base at each step must be tracked, not the original wholesale cost reused.

Route: If check passes: move to PR-PC-14. If check fails: reteach with the base-tracking strip mandatory on every multi-step item.

PR-PC-14 Recognize successive percent changes combine multiplicatively

Mathematical claim: Two successive percent changes do not combine by adding the percents; equal-percent opposite changes do not return to the original amount.

Prerequisites: PR-PC-13

Route Finder

Item: A price increases 20%, then decreases 20%. Is the final price the same as the original?

Correct response: No -- lower. $\$100 \rightarrow \$120 \rightarrow \$96$, not back to \$100.

Tripwire

Observed: A price increases 25% then decreases 25%, and a student assumes it returns to the original value.

Likely wrong response: Assumes the two changes cancel out and return to \$100.

Correct response: No -- $\$100 \rightarrow \$125 \rightarrow \$93.75$, lower than the original \$100.

Likely reasoning: may indicate equal-percent changes in opposite directions are being assumed to cancel out and return to the starting value

Confirm (approach): Give a second equal-opposite-percent item with different numbers, and require the intermediate value shown. If the cancel-out assumption recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A price drops 30% then rises 30%. Is the result the original price? Show the intermediate value.

Correct response: No -- lower. $\$100 \rightarrow \$70 \rightarrow \$91$, not $\$100$.

Teach: Base-tracking table showing the intermediate value explicitly between the two steps, making the mismatch with the original visible.

Fresh Check (Recalculation Check)

Item: A price decreases 10% then increases 10%. Is the result back to the original? Show the intermediate value.

Correct response: No -- lower than original. $\$90 \rightarrow \$99$, not $\$100$.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A stock loses 50% of its value one year, then gains 50% the next, described purely as an investment story with no framing about combining percents.

Correct response: Not back to even -- $\$100 \rightarrow \$50 \rightarrow \$75$, a net loss of 25%.

Route: If check passes: move to PR-PC-15. If check fails: reteach with the intermediate-value table until the non-cancellation is reliably shown.

PR-PC-15 Apply and distinguish specialized percent contexts

Mathematical claim: Simple interest applies the same rate to the original principal each period; a reverse-percent problem (given the result, find the original) requires dividing, not multiplying.

Prerequisites: PR-PC-13

Route Finder

Item: A \$500 result came from applying a 25% increase to an unknown original amount. Find the original amount.

Correct response: \$400 ($\$500 \div 1.25 = \$400$).

Tripwire

Observed: A result of \$260 came from a 30% increase on an unknown original. A student multiplies \$260 by 1.30 instead of dividing.

Likely wrong response: \$338, from applying the forward (multiply) operation instead of reversing it.

Correct response: \$200 ($\$260 \div 1.30 = \$200$).

Likely reasoning: may indicate the forward operation is being applied instead of being reversed for a reverse-percent problem

Confirm (approach): Give a second reverse-percent item alongside a forward item using similar numbers, to check whether direction is reasoned about. If the forward operation is applied again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: After a 4% annual simple interest rate for 3 years, a \$1000 deposit earns how much total interest?

Correct response: \$120 ($1000 \times 0.04 \times 3$ -- same rate on the same original principal each year).

Teach: Explicit reverse-operation check: “what operation undoes this step?” before solving a reverse-percent problem.

Fresh Check (Recalculation Check)

Item: A result of \$345 came from a 15% increase on an unknown original. Find the original.

Correct response: \$300 ($\$345 \div 1.15$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A final price of \$172.50 already includes a 15% tip. What was the bill before the tip, asked with no reverse-percent framing?

Correct response: \$150 ($\$172.50 \div 1.15$).

Route: If check passes: chapter secure, move to PR-RR-04 (Chapter 9). If check fails: reteach the reverse-operation check as mandatory before any ‘find the original’ item.

Chapter 9 -- Ratios: Part-to-Part and Part-to-Whole

Chapter Support

Materials: Prepare red and blue counters, linking cubes, labeled quantity cards, tape-diagram strips, ratio tables, fraction cards, and context cards. Keep color or symbol labels on all materials so students do not detach numbers from quantities.

Differentiation

Students needing concrete support

Use colored counters and physically build every ratio. Do not remove labels. Move from 2:3 and 1:4 to less familiar ratios only after students can state both the part-to-part and part-to-whole meanings.

Students needing language support

Use the frames:

“The ratio compares _____ with _____.”

“The first number represents _____.”

“The total number of parts is _____.”

“The fraction of the whole is _____ because _____.”

“One ratio part is _____, so the total quantities are _____.”

Students ready to extend

Ask students to construct multiple contexts for the same ratio, compare ratios with the same part-to-whole fraction, solve missing-total problems, and explain why ratios can be equivalent even when their numbers differ.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given a part-to-part ratio, write both corresponding part-to-whole fractions, and explain why they must add to 1.
2. Create a ratio where reversing the order changes which real-world quantity is described first, and explain the practical difference.
3. Simplify a ratio using a common factor that is not the greatest common factor, then show the fully simplified form and explain what was missed.

4. A tape diagram is drawn with unequal-sized boxes for a ratio. Explain what's wrong with the diagram and how to fix it.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-04 Distinguish part-to-part ratio from part-to-whole fraction

Mathematical claim: A ratio like 2:3 compares two parts; converting to part-to-whole fractions requires adding both parts to find the whole, not treating one part or the second term as the whole.

Prerequisites: PR-FR-01

Route Finder

Item: A group has a ratio of cats to dogs of 3:5. What fraction of the group are cats?

Correct response: $\frac{3}{8}$ (whole = 3+5 = 8 parts).

Tripwire

Observed: A ratio of red to blue marbles is 2:7, and a student states that red marbles are $\frac{2}{7}$ of the total.

Likely wrong response: $\frac{2}{7}$, treating the second ratio term as if it were already the whole.

Correct response: $\frac{2}{9}$ (whole = 2+7 = 9 parts).

Likely reasoning: may indicate the second ratio term, or one part, is being treated as the whole, skipping the addition step

Confirm (approach): Give a second part-to-part-to-fraction conversion with different numbers. If the same skipped-addition pattern recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A recipe ratio of butter to sugar is 4:5. What fraction of the mixture (by these two ingredients only) is sugar?

Correct response: $\frac{5}{9}$ (whole = $4+5 = 9$ parts).

Teach: Labeled tape diagram. Talk Prompt: “Are you comparing one part with another part, or a part with the whole?” Require the addition step shown before any fraction is written.

Fresh Check (Recalculation Check)

Item: A ratio of boys to girls is 4:9. What fraction of the group is boys?

Correct response: $\frac{4}{13}$ (whole = $4+9 = 13$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A classroom has a ratio of 5 windows to 3 doors, mentioned in passing while describing the room, with no ratio-to-fraction instruction.

Correct response: Windows are $\frac{5}{8}$ of the windows-and-doors total (whole = $5+3=8$).

Route: If check passes: move to PR-RR-05. If check fails: reteach with the tape diagram, requiring the total-parts addition shown explicitly every time.

PR-RR-05 Track which ratio number corresponds to which label

Mathematical claim: A ratio's order encodes meaning; reversing stated order swaps which quantity comes first, and this must be tracked explicitly under rescaling or reversal.

Prerequisites: PR-RR-04

Route Finder

Item: A recipe ratio of flour to sugar is 3:2. If reversed to sugar:flour, what is the new ratio?

Correct response: 2:3.

Tripwire

Observed: A ratio of red paint to blue paint is 5:3. Asked for the ratio of blue to red, a student answers 5:3 again.

Likely wrong response: 5:3, not adjusting for the reversal of which quantity is named first.

Correct response: 3:5.

Likely reasoning: may indicate which ratio number corresponds to which label is being lost track of under reversal

Confirm (approach): Give a second item with a ratio reversed partway through a multi-part question, checking whether label tracking holds. If tracking is lost again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A fruit basket ratio of apples to oranges is 6:4. State the ratio of oranges to apples.

Correct response: 4:6 (or simplified 2:3).

Teach: Tape diagram with each box explicitly labeled by name or color, re-labeled visibly upon any reversal.

Fresh Check (Recalculation Check)

Item: A ratio of apples to oranges is 7:4. What is the ratio of oranges to apples?

Correct response: 4:7.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A survey reports a 7:9 ratio of yes to no votes on a proposal. A follow-up question asks for the no-to-yes ratio, without using the word 'reverse.'

Correct response: 9:7.

Route: If check passes: move to PR-RR-06. If check fails: reteach with physical, re-labelable counters for every reversal.

PR-RR-06 Simplify a ratio using the greatest common factor

Mathematical claim: Simplifying a ratio requires dividing both parts by their greatest common factor; stopping at a smaller common factor leaves it incompletely simplified.

Prerequisites: PR-RR-04

Route Finder

Item: Simplify the ratio 18:24 completely.

Correct response: 3:4 (GCF of 18 and 24 is 6).

Tripwire

Observed: A student simplifies 16:24 to 8:12 and stops there.

Likely wrong response: 8:12, having used a common factor (2) smaller than the greatest one (8).

Correct response: 2:3 (GCF of 16 and 24 is 8, not 2).

Likely reasoning: *may indicate a smaller common factor is being used instead of the greatest one, leaving the ratio only partially simplified*

Confirm (approach): Give a second ratio with a GCF larger than 2, checking whether full simplification is reached. If it stops early again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Simplify the ratio 28:42 to its lowest terms.

Correct response: 2:3 (GCF of 28 and 42 is 14).

Teach: Verify both parts share the same single greatest factor before accepting a simplified form as final.

Fresh Check (Recalculation Check)

Item: Simplify 20:32 completely.

Correct response: 5:8 (GCF is 4).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A survey of 36:54 people prefer two brands, mentioned in a marketing report with no instruction to simplify.

Correct response: 2:3 (GCF is 18).

Route: If check passes: chapter secure, move to PR-RR-07 (Chapter 10). If check fails: reteach by listing all common factors and selecting the greatest, every time.

Chapter 10 -- Equivalent Ratios and Ratio Tables

Chapter Support

Materials: Prepare two-color counters, grouped strips, recipe cards, scale-factor cards, ratio-table templates, double number lines, and context cards. Use physical grouping before introducing abstract completion of $a:b = c:d$.

Differentiation

Students needing concrete support

Use equal groups of counters and physical recipe cards. Keep the original ratio visible, and create one full group at a time. Use scale factors 2, 3, 4, and 5 before fractional or reverse scaling.

Students needing language support

Use the frames:

“The original ratio compares _____ with _____.”

“The scale factor is _____ because _____.”

“I multiply both parts by _____.”

“The ratios are equivalent because _____.”

“The relationship changes when _____.”

Students ready to extend

Ask students to scale by fractions, compare multiple equivalent tables, analyze maps and mixtures, and prove equivalence by simplification, scale factor, and double-number-line reasoning.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Create two ratios that are additively related (same difference added to both parts) but not multiplicatively equivalent, and explain the distinction.
2. Given a ratio and a claimed equivalent ratio, verify equivalence by checking both parts' scale factors, not just one.
3. Scale a ratio by a non-whole-number factor and explain why the result is still a valid equivalent ratio.
4. Build a ratio table with four rows where one row secretly uses a different scale factor, and explain how to catch the error.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-07 Recognize equivalent ratios are related multiplicatively

Mathematical claim: Two ratios are equivalent only if the same multiplicative factor connects both corresponding parts; an additive shift does not preserve the ratio. Dominant error in this chapter, by a wide margin.

Prerequisites: PR-RR-05

Route Finder

Item: Is 3:5 equivalent to 9:15?

Correct response: Yes -- both parts scaled by 3.

Tripwire

Observed: A student is asked whether 4:7 is equivalent to 7:10, reasoning “yes, because you add 3 to both.”

Likely wrong response: “Yes,” from an additive shift (+3 to both parts) rather than checking a multiplicative factor.

Correct response: No -- 4:7 scaled by any single factor does not give 7:10 ($4 \times k = 7$ gives $k = 1.75$, but $7 \times 1.75 = 12.25 \neq 10$). Not equivalent.

Likely reasoning: may indicate an additive shift is being used to judge or produce equivalence rather than a multiplicative scale factor, the chapter’s dominant error

Confirm (approach): Give a second pair that is additively but not multiplicatively related, and a third pair that is genuinely equivalent, to separate the two. If the additive pattern recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Is a mixing ratio of 2:9 equivalent to 8:36? Justify with a scale factor, not addition.

Correct response: Yes -- both scaled by 4.

Teach: Ratio table with an explicit scale-factor column. Talk Prompt: “What factor changed the first part? Does the second part use that same factor?”

Fresh Check (Recalculation Check)

Item: Is 6:8 equivalent to 15:20?

Correct response: Yes -- both scaled by 2.5.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A paint mix uses 3 parts blue to 8 parts white. Someone suggests 9 parts blue to 14 parts white makes “the same shade,” with no equivalence language used.

Correct response: Not the same shade -- 3:8 scaled would need 9:24, not 9:14; the suggested mix is not equivalent.

Route: If check passes: move to PR-RR-08. If check fails: reteach with the scale-factor column mandatory on every equivalence check.

PR-RR-08 Verify one consistent scale factor applies to both parts

Mathematical claim: Confirming equivalence requires checking that the same factor connects both parts, not just one.

Prerequisites: PR-RR-07

Route Finder

Item: Is 4:9 equivalent to 12:27? Show the factor check for both parts.

Correct response: Yes -- both parts $\times 3$.

Tripwire

Observed: A student checks whether 5:8 is equivalent to 15:20 by confirming $5 \times 3 = 15$, and concludes “yes” without checking the second part.

Likely wrong response: “Yes,” from checking only the first part’s factor and assuming the second matches.

Correct response: No -- $8 \times 3 = 24 \neq 20$, so the ratios are not equivalent.

Likely reasoning: may indicate only one term’s scale factor is being checked, with the other assumed to match without verification

Confirm (approach): Give a second pair where one part’s factor matches but the other doesn’t, deliberately designed to pass a one-term check. If the same partial check recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Is 6:11 equivalent to 18:34? Check both parts’ factors explicitly.

Correct response: No -- $6 \times 3 = 18$ matches, but $11 \times 3 = 33 \neq 34$.

Teach: Require the factor stated explicitly for both parts before accepting an equivalence claim as verified.

Fresh Check (Recalculation Check)

Item: Is 7:6 equivalent to 21:20? Check both parts’ factors.

Correct response: No -- $7 \times 3 = 21$ matches, but $6 \times 3 = 18 \neq 20$.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe scaled from 4:9 to 12:26 is questioned by a reader who “thinks something's off,” with no factor-checking instruction given.

Correct response: Something is off -- $4 \times 3 = 12$ matches, but $9 \times 3 = 27 \neq 26$.

Route: If check passes: move to PR-RR-09. If check fails: reteach requiring both factor checks shown on every equivalence item.

PR-RR-09 Explain why matching simplest forms confirms equivalence

Mathematical claim: Two ratios are equivalent if and only if they reduce to the same simplest form; this is the actual test of equivalence, not just a final tidying step.

Prerequisites: PR-RR-08

Route Finder

Item: Are 8:12 and 10:15 equivalent? Use simplest form to justify.

Correct response: Yes -- both simplify to 2:3.

Tripwire

Observed: A student is asked why simplifying both ratios to 3:4 confirms they are equivalent, and answers “because you simplify them,” without explaining what matching simplest forms actually shows.

Likely wrong response: Restates that simplification was performed without explaining why matching results confirm equivalence.

Correct response: Matching simplest forms means both ratios describe the same underlying relationship once all common factors are removed -- that shared reduced form is the equivalence test itself.

Likely reasoning: *may indicate the connection between matching simplest forms and the definition of equivalence has not been stated, only the mechanical step performed*

Confirm (approach): Give a second explain-why item in a different numeric context. If the explanation is skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Are 14:21 and 20:30 equivalent? Simplify both and explain what matching forms shows.

Correct response: Yes -- both simplify to 2:3, confirming the same underlying relationship.

Teach: Require the connecting sentence linking simplest form explicitly to the equivalence test, not just the simplified numbers.

Fresh Check (Recalculation Check)

Item: Are 9:15 and 12:20 equivalent? Simplify and explain why matching forms confirms it.

Correct response: Yes -- both simplify to 3:5; matching simplest forms means the same relationship, confirming equivalence.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two recipe cards list flour-to-water ratios of 10:15 and 8:12, and a baker wonders if they're really the same recipe, with no simplification instruction given.

Correct response: Yes, the same ratio -- both simplify to 2:3.

Route: If check passes: chapter secure, move to PR-RR-10 (Chapter 11). If check fails: reteach requiring the explanatory sentence on every equivalence item.

Chapter 11 -- Rates and Unit Rates

Chapter Support

Materials: Prepare item cards, price cards, distance and time strips, page cards, two-column rate tables, unit labels, and double number lines. Write the requested units before students calculate.

Differentiation

Students needing concrete support

Use small whole-number examples and physically distribute the total among equal groups. Keep the unit card attached to each group. Move from money to speed and pages only after the meaning of “per one” is secure.

Students needing language support

Use the frames:

“The rate compares _____ with _____.”

“The requested unit rate is _____ per _____.”

"I divide _____ by _____ because _____."

"The unit rate is _____."

"The answer is reasonable because _____ $\times$ _____ = _____."

Students ready to extend

Use decimal and fractional quantities, unit conversions, rates per minute versus per hour, and inverse problems where the unit rate is given and a total must be found.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given a rate in one direction (km per hour), find the equivalent rate in the reverse direction (hours per km), and explain when each form is useful.
2. Create two situations with the same numbers where one calls for scaling a rate and the other for combining two separate quantities, and solve both.
3. A unit rate calculation gives a numerically correct answer with no units attached. Explain why this is an incomplete answer.
4. Compare two rates given in different units (e.g., mL per second vs. L per minute) and explain the conversion needed before comparing.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-10 Divide in the direction matching the requested unit

Mathematical claim: A unit rate is found by dividing so the answer lands in the requested unit; dividing in reverse produces a meaningless value for the question asked. Dominant error in this chapter.

Prerequisites: PR-FR-06

Route Finder

Item: A car travels 180 km in 3 hours. Find the speed in km per hour.

Correct response: 60 km/h ($180 \div 3$).

Tripwire

Observed: A recipe uses 5 cups of flour for 20 servings. Asked for cups per serving, a student computes $20 \div 5 = 4$.

Likely wrong response: 4, from dividing in the direction that doesn't match the requested unit (servings per cup instead of cups per serving).

Correct response: 0.25 cups per serving ($5 \div 20$).

Likely reasoning: may indicate division is being performed in the direction that doesn't match the unit the question actually requests, the chapter's dominant error

Confirm (approach): Give a second rate item where both division directions produce plausible-looking numbers, forcing a units check. If the wrong direction recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A printer produces 250 pages in 10 minutes. State the rate in pages per minute.

Correct response: 25 pages per minute.

Teach: Rate table with a labeled one-unit row. Talk Prompt: "What should the answer say -- per one what?" Require the unit stated before dividing.

Fresh Check (Recalculation Check)

Item: A pump fills 150 liters in 6 minutes. Find the rate in liters per minute.

Correct response: 25 liters per minute.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A well pumps 180 liters over 12 minutes, mentioned in a maintenance log with no unit-rate instruction.

Correct response: 15 liters per minute.

Route: If check passes: move to PR-RR-11. If check fails: reteach requiring the target unit named in words before any division is performed.

PR-RR-11 Compare situations using unit rates, not raw totals

Mathematical claim: Comparing which of two options is more requires unit rates, not raw totals, whenever the underlying quantities differ in size.

Prerequisites: PR-RR-10

Route Finder

Item: Store A: 10 pens for \$8. Store B: 15 pens for \$10.50. Which has the lower unit price?

Correct response: Store B (\$0.70/pen vs. Store A's \$0.80/pen).

Tripwire

Observed: Store A sells 6 apples for \$3. Store B sells 10 apples for \$4.50. A student says Store A is cheaper because \$3 is less than \$4.50.

Likely wrong response: "Store A is cheaper," comparing the raw totals (\$3 vs. \$4.50) without computing unit prices.

Correct response: Store B has the lower unit price (\$0.45/apple vs. Store A's \$0.50/apple).

Likely reasoning: *may indicate totals are being compared directly without accounting for the different quantities each total represents*

Confirm (approach): Give a second comparison item where the option with the larger total actually has the better unit rate. If totals are compared directly again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Brand X: 8 batteries for \$10. Brand Y: 12 batteries for \$16.20. Which has the lower unit price?

Correct response: Brand X (\$1.25/battery vs. Brand Y's \$1.35/battery).

Teach: Rate table showing both totals and the derived unit rate side by side for direct comparison.

Fresh Check (Recalculation Check)

Item: Store A: 8 notebooks for \$12. Store B: 5 notebooks for \$8. Which is the better unit price?

Correct response: Store A has the better price: \$1.50/notebook vs. Store B's \$1.60/notebook.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A shopper sees a 3-pack of soap for \$4.50 and a 5-pack for \$7.25, and wonders which is the better deal, with no mention of unit rates.

Correct response: The 5-pack (\$1.45/bar vs. the 3-pack's \$1.50/bar).

Route: If check passes: move to PR-RR-12. If check fails: reteach requiring the unit rate computed for each option before any comparison is stated.

PR-RR-12 Distinguish scaling a rate from combining quantities

Mathematical claim: Scaling a rate to a new total is multiplicative, distinct from combining two separate quantities by addition.

Prerequisites: PR-RR-10

Route Finder

Item: A recipe uses 2 cups flour for 8 servings. How much flour for 20 servings?

Correct response: 5 cups (unit rate $0.25 \text{ cups/serving} \times 20$).

Tripwire

Observed: A pump fills 15 liters in 3 minutes. Asked how much it fills in 3 more minutes plus the original, a student adds $15+3=18$ liters.

Likely wrong response: 18, from combining the original amount with the elapsed time rather than scaling the rate to the new total time.

Correct response: 30 liters total after 6 minutes (rate is $5 \text{ L/min} \times 6 \text{ minutes}$), not $15+3$.

Likely reasoning: may indicate scaling is being confused with combining, applying addition where multiplication by the rate is required

Confirm (approach): Give a second scaling item where the additive combination produces a plausible but wrong answer. If combining recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A conveyor belt moves 24 boxes in 4 minutes at a constant rate. How many boxes in 15 minutes total?

Correct response: 90 boxes (rate $6 \text{ boxes/min} \times 15$).

Teach: Rate table applying the same per-unit rate explicitly to the new total, shown as multiplication, not addition.

Fresh Check (Recalculation Check)

Item: A hose fills 40 liters in 5 minutes. How much does it fill in 12 minutes?

Correct response: 96 liters (rate $8 \text{ L/min} \times 12$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A factory produces 40 units in 8 hours. How many in 20 hours, phrased in a production report with no rate language used?

Correct response: 100 units ($5 \text{ units/hour} \times 20$).

Route: If check passes: chapter secure, move to PR-RR-13 (Chapter 12). If check fails: reteach distinguishing scaling from combining with concrete rate-table examples.

Chapter 12 -- Comparing Rates and Best Buys

Chapter Support

Materials: Prepare package cards, price cards, 100 mL and 100 g strips, litre and kilogram conversion cards, double number lines, unit-price grids, and decision sentence frames.

Differentiation

Students needing concrete support

Use package strips divided into equal unit boxes. Put the price above the strip and physically distribute the price among the boxes. For volume, place 100 mL labels on both bottles before calculating.

Students needing language support

Use the frames:

“The common comparison unit is _____.”

“Option A costs/is _____ per _____.”

“Option B costs/is _____ per _____.”

“Option _____ is better mathematical value because _____.”

“A different practical choice might be reasonable because _____.”

Students ready to extend

Use fractional unit rates, unit conversions involving minutes and hours, equal-value cases, and comparisons in which the mathematically better package is not the practical choice.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given two deals with different package sizes and prices, determine the better value and explain why comparing totals directly would be misleading.

2. Create a scenario where the larger total price is actually the better deal, and explain what makes this case different from usual.
3. Compare two rates given in incompatible units and identify the conversion needed before any comparison is valid.
4. State a complete best-buy conclusion that includes both the computed unit rates and the final recommendation as separate parts.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-13 Compare rates in compatible units, converting first if needed

Mathematical claim: Comparing two rates requires the same units on both sides; a larger package or more items is not itself evidence of better value.

Prerequisites: PR-RR-11

Route Finder

Item: Brand A: 500 g for \$4. Brand B: 1.2 kg for \$9. Which is the better price per gram?

Correct response: Brand A: \$0.008/g. Brand B ($\$ \frac{9}{1200} \text{g} = \$0.0075/\text{g}$) is slightly better.

Tripwire

Observed: A student compares a 12-pack of juice boxes for \$6 to a single 2-liter bottle for \$3, concluding the 12-pack is the better deal because it has more items.

Likely wrong response: Concludes based on item count or package size alone, without converting to a common unit and computing.

Correct response: Depends on the actual volume per dollar in a common unit -- “more items” alone does not establish better value without converting to a shared unit and computing the rate.

Likely reasoning: *may indicate a larger package or more items is being assumed automatically better without computing the unit rate in a common unit*

Confirm (approach): Give a second item in incompatible units (e.g., grams vs. kilograms) where the uncomputed larger number is misleading. If the assumption recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Cereal A: 340 g for \$3.20. Cereal B: 500 g for \$4.80. Which has the lower price per gram?

Correct response: Cereal A: \$0.0094/g. Cereal B: \$0.0096/g -- Cereal A is slightly lower.

Teach: Comparison table with both unit rates converted to the same unit, shown side by side.

Fresh Check (Recalculation Check)

Item: Brand A: 750 mL for \$5. Brand B: 1 L for \$6.40. Which is the better price per mL?

Correct response: Brand A: \$0.00667/mL. Brand B: \$0.0064/mL -- Brand B is slightly better.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A shopper compares a 750 mL bottle at \$2.10 to a 1.5 L bottle at \$3.90, without being told to convert units first.

Correct response: 750 mL bottle: \$0.0028/mL. 1.5 L (1500 mL) bottle: \$0.0026/mL -- the larger bottle is slightly better once converted to the same unit.

Route: If check passes: move to PR-RR-14. If check fails: reteach unit conversion as a mandatory first step before any rate comparison.

PR-RR-14 State both the unit rates and the conclusion separately

Mathematical claim: A best-buy answer needs the unit rate for each option (the math) and the conclusion (the choice) stated as two distinct, explicit parts.

Prerequisites: PR-RR-13

Route Finder

Item: Compare 4 notebooks for \$20 to 6 notebooks for \$24, stating both unit rates and your conclusion.

Correct response: \$5/notebook vs. \$4/notebook; the second option is the better deal.

Tripwire

Observed: A student is asked to compare two deals and answers only “the second one is better” with no unit rates shown.

Likely wrong response: States only the conclusion, or only the unit rates, without connecting the two explicitly.

Correct response: A complete answer states both computed unit rates and then the conclusion they support.

Likely reasoning: *may indicate a conclusion is being stated without the unit rates that justify it, or vice versa*

Confirm (approach): Give a second comparison item, scoring the answer as incomplete if either part is missing. If one part is dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Compare 7 markers for \$10.50 to 12 markers for \$16.80, stating both unit rates and your conclusion.

Correct response: \$1.50/marker vs. \$1.40/marker; the second option is the better deal.

Teach: Require both the “what one unit could both options share?” computation and a separate, explicit concluding sentence.

Fresh Check (Recalculation Check)

Item: Compare 3 shirts for \$27 to 5 shirts for \$40, stating both unit rates and your conclusion.

Correct response: \$9/shirt vs. \$8/shirt; the second option is the better deal.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A parent is choosing between two multi-packs of juice boxes and asks a friend which is the better buy, and the friend just says “the bigger one,” with no rates mentioned.

Correct response: A complete answer requires both unit rates stated, not just a size-based guess.

Route: If check passes: chapter secure, move to PR-RR-15 (Chapter 13). If check fails: reteach requiring both parts on every comparison item, scored as incomplete otherwise.

Chapter 13 -- Solving Proportions with More Than One Strategy

Chapter Support

Materials: Prepare two-color counters, ratio cards, tape strips, ratio tables, unit-rate cards, equation cards, cross-product arrows, map-scale cards, and “proportional or not?” situation cards.

Differentiation

Students needing concrete support

Use two-color ratio groups and physically copy groups to show a scale factor. Keep numerator and denominator labels attached to the counters. Move to tables only after students can explain why both quantities must be scaled by the same factor.

Students needing language support

Use the frames:

"The first ratio compares _____ with _____."

"The scale factor is _____ because _____."

"I chose _____ because _____."

"The ratio order stays the same: _____ over _____."

"I checked by _____."

Students ready to extend

Require two methods for the same problem, ask students to construct non-proportional counterexamples, and include decimal solutions and unit conversions.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Solve the same proportion using two different methods (cross-products and unit rate) and confirm the answers agree.
2. Create a proportion where cross-multiplying non-corresponding terms gives a plausible but wrong answer, and explain the correct pairing.
3. Explain, with a specific example, why choosing a solving method should depend on the numbers given, not personal preference alone.
4. Justify why a specific relationship is not proportional using the zero-input test, not just a general claim.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-15 Solve a proportion using cross-products correctly

Mathematical claim: Cross-products multiplies diagonally corresponding terms; the numerator of each ratio must represent the same kind of quantity.

Prerequisites: PR-RR-07

Route Finder

Item: Solve for x: $\frac{3}{4} = x/20$.

Correct response: $x = 15$ ($3 \times 20 = 4 \times x$, so $60 = 4x$, $x = 15$).

Tripwire

Observed: Solving $\frac{2}{5} = x/15$, a student sets up $2 \times x = 5 \times 15$ (multiplying non-diagonal terms).

Likely wrong response: An incorrect equation from multiplying terms that are not actually diagonal to each other.

Correct response: $x = 6$ ($2 \times 15 = 5 \times x$, so $30 = 5x$, $x = 6$).

Likely reasoning: may indicate cross-products are being computed by multiplying terms that are not diagonally corresponding

Confirm (approach): Give a second proportion with the unknown in a different position, checking whether the correct diagonal pairing transfers. If the wrong pairing recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve for x: $\frac{7}{9} = x/36$.

Correct response: $x = 28$ ($7 \times 36 = 9 \times x$, $252 = 9x$, $x = 28$).

Teach: Four-method decision map, cross-products branch, with numerator roles labeled explicitly before multiplying.

Fresh Check (Recalculation Check)

Item: Solve for x: $\frac{5}{8} = x/24$.

Correct response: $x = 15$.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A scale drawing uses the relationship $\frac{5}{8} = x/40$ to find a missing length, presented as a drafting problem with no 'cross-products' instruction.

Correct response: $x = 25$.

Route: If check passes: move to PR-RR-16. If check fails: reteach labeling the diagonal pairs explicitly before every cross-multiplication.

PR-RR-16 Choose a strategy and reconcile results when methods disagree

Mathematical claim: A proportion can be solved by more than one valid method; if two methods disagree, that signals an error to find, not one to leave unresolved.

Prerequisites: PR-RR-15

Route Finder

Item: Solve $\frac{4}{6} = x/9$ using both the scale-factor method and cross-products. Do they agree?

Correct response: $x=6$ both ways: scale factor 1.5 ($4 \times 1.5=6$); cross-products $4 \times 9=6 \times x$, $36=6x$, $x=6$.

Tripwire

Observed: A student solves a proportion by unit rate (getting 12) and by cross-products (getting 15), and submits 15 without addressing the mismatch.

Likely wrong response: Submits one of the two disagreeing answers without resolving the mismatch.

Correct response: Correct final answer should be determined by finding and fixing the error in one of the two methods, not by picking one arbitrarily.

Likely reasoning: *may indicate a mismatch between two solving methods is going unnoticed or unreconciled*

Confirm (approach): Give a second item with an intentional error planted in one method, checking whether the mismatch is caught. If it goes unnoticed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve $\frac{9}{12} = x/16$ using two different methods. Do they agree?

Correct response: $x=12$ both ways: scale factor $\frac{4}{3}$ ($9 \times \frac{4}{3}=12$); cross-products $9 \times 16=12 \times x$, $144=12x$, $x=12$.

Teach: Four-method decision map, requiring a stated reason for the chosen method and both methods' results compared when time allows.

Fresh Check (Recalculation Check)

Item: Solve $\frac{6}{10} = x/25$ two ways and confirm they agree.

Correct response: $x = 15$ both ways.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe conversion gives two different answers (14 and 16) for the same missing quantity using two methods, and a cook wants to know which to trust.

Correct response: The mismatch itself signals an error in one method; it must be found and fixed, not resolved by picking either number.

Route: If check passes: move to PR-RR-17. If check fails: reteach requiring a second method as a check on every proportion until mismatches are reliably caught.

PR-RR-17 Justify non-proportionality with specific named evidence

Mathematical claim: Showing a relationship is not proportional requires naming the specific evidence (a nonzero starting amount, a changing rate, or a failed zero-input test), not just asserting it.

Prerequisites: PR-RR-07

Route Finder

Item: A cost starts at \$5 with no items and increases by \$3 per item. Is this proportional? Justify.

Correct response: No -- at 0 items the cost is \$5, not \$0 (fails the zero-input test).

Tripwire

Observed: A student is shown a relationship with a nonzero starting value and states “it’s not proportional” with no further explanation.

Likely wrong response: States the conclusion (“not proportional”) without naming the specific disqualifying evidence.

Correct response: The specific evidence must be named: e.g., the value at zero input is nonzero, or the rate changes across the table.

Likely reasoning: may indicate the specific evidence for non-proportionality is not being identified, only the classification

Confirm (approach): Give a second non-proportional item with different evidence (a changing rate rather than a nonzero start), checking whether the specific reasoning is named. If evidence is omitted again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A cost starts at \$12 with 0 uses and stays exactly \$12 regardless of use count. Is this proportional? Name your evidence.

Correct response: No -- the value is nonzero at 0 uses (\$12), so it fails the zero-input test even though the rate of change is technically 0, not varying.

Teach: Require the zero-input or changing-rate evidence stated explicitly, not just the yes/no classification.

Fresh Check (Recalculation Check)

Item: A relationship has values (1,7), (2,15), (3,25). Is this proportional? Name your evidence.

Correct response: No -- the quotient changes (7, 7.5, 8.33...), not constant.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A subscription service charges the same amount every month regardless of usage, and a customer wonders if cost is proportional to usage, with no zero-input framing given.

Correct response: No -- cost at 0 usage is still the flat fee, not \$0, so it's not proportional to usage.

Route: If check passes: chapter secure, move to PR-RR-18 (Chapter 14). If check fails: reteach requiring named evidence on every non-proportionality claim.

Chapter 14 -- Proportional Relationships in Tables, Graphs, and Equations

Chapter Support

Materials: Prepare input/output cards, unit-rate strips, coordinate grids, graph paper, equation cards, tables with missing entries, origin stickers, and proportional/non-proportional relationship cards.

Differentiation

Students needing concrete support

Use quantity-pair cards and build tables from physical groups. Put an origin card beside the “zero input” situation. Delay formal equation writing until students can state the unit rate in words.

Students needing language support

Use the frames:

“The input is _____ and the output is _____.”

“The constant of proportionality is _____ because _____ $\div$ _____ = _____.”

“The equation is _____.”

“The graph is/is not proportional because _____.”

“At zero input, the output should be _____.”

Students ready to extend

Use fractional constants, negative examples, tables with missing values, graph interpretation, and models with starting amounts. Ask students to write two equations that have the same rate but different intercepts and explain the difference.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given a table, a graph, and an equation, identify which two do not represent the same proportional relationship and explain how you found the mismatch.
2. Create a table with a constant difference but a non-constant quotient, and explain why ‘the pattern goes up steadily’ is not sufficient evidence of proportionality.
3. Build one proportional and one non-proportional equation that look superficially similar, and verify both using the origin test.
4. Given only a graph passing through the origin, determine what additional check is needed before concluding the relationship is proportional.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-18 Test any representation for proportionality using the same two checks

Mathematical claim: A table, graph, or equation is proportional only if the quotient is constant throughout AND the origin is included. An increasing pattern alone is not sufficient. This is Chapter 1's core confusion resurfacing at a more advanced representational level.

Prerequisites: PR-RR-01

Route Finder

Item: A graph shows a line increasing steadily but crossing the vertical axis at (0,4), not the origin. Is this relationship proportional?

Correct response: No -- it does not pass through (0,0), regardless of the constant rate of increase.

Tripwire

Observed: A table shows (1,6), (2,10), (3,14). A student says this is proportional because the values keep increasing steadily.

Likely wrong response: "Yes, proportional," from the pattern increasing steadily (addition is enough reasoning).

Correct response: No -- the difference is constant (4) but the quotient is not (6, 5, 4.67...), and it does not pass through (0,0) given the pattern (value at 0 would be 2, not 0).

Likely reasoning: may indicate a constant difference or a generally increasing pattern is being mistaken for the proportional relationship, without the quotient and origin checks -- the same confusion first seen in PR-RR-01, now at the table/graph/equation level

Confirm (approach): Give a second item in a different representation (graph or equation) with the same additive-pattern trap. If the same error recurs across representations, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A graph passes through (0,0) and (5,5), (10,8). Is this relationship proportional? Justify with both checks.

Correct response: No -- although it passes through the origin, the quotient is not constant ($\frac{5}{5}=1$, $\frac{8}{10}=0.8$).

Teach: Table-graph-equation translation map. Talk Prompt: "Does the output divided by input stay constant? What happens at zero input?" Require both checks shown, not just one.

Fresh Check (Recalculation Check)

Item: An equation is $y = 3x + 2$. Proportional or not? Justify using both checks.

Correct response: Not proportional -- at $x=0$, $y=2$, not 0.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A phone plan's cost table shows (1,20),(2,40),(3,60), presented as a billing table with no mention of proportionality testing.

Correct response: Proportional -- constant quotient of 20, and \$0 at 0 minutes is consistent.

Route: If check passes: move to PR-RR-19. If check fails: reteach by returning to PR-RR-01's double number line before continuing at the graph/equation level.

PR-RR-19 Identify the same constant k across table, graph, and equation

Mathematical claim: When one proportional relationship is shown in multiple representations, the same constant k must be identifiable in each: the table's quotient, the graph's steepness, the equation's coefficient.

Prerequisites: PR-RR-18

Route Finder

Item: A table shows (2,10), (4,20). The equation is $y=5x$. Identify k in both and confirm they match.

Correct response: $k=5$ in both -- the table's quotient ($\frac{10}{2}=5$) matches the equation's coefficient.

Tripwire

Observed: Given a table, a graph, and an equation of one proportional relationship, a student is asked what must be true across all three, and answers "they should all look similar" without naming the shared constant.

Likely wrong response: A vague answer that doesn't name the constant k specifically as the shared, checkable feature.

Correct response: The same constant k must be identifiable in the table's quotient, the graph's steepness, and the equation's coefficient.

Likely reasoning: *may indicate the three representations are understood as generally related without the specific shared constant k being named*

Confirm (approach): Give a second cross-representation item and require k named and located in all three. If the naming is skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A table shows (4,36) and a graph passes through (1,9). Are these the same proportional relationship? Identify k in each.

Correct response: Yes -- $k=9$ in both ($\frac{36}{4}=9$, and the graph point gives $\frac{9}{1}=9$).

Teach: Translation map with k explicitly labeled in each representation, side by side.

Fresh Check (Recalculation Check)

Item: A table shows (3,21), a graph passes through (1,7), and an equation is given as $y=7x$. Confirm k matches across all three.

Correct response: $k=7$ in all three.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A recipe's cost is described in a table, a graph, and a formula, and a reader asks whether all three describe the same situation, with no mention of a shared constant.

Correct response: They match only if the same constant (the cost per unit) appears in all three; that shared value is what must be checked.

Route: If check passes: move to PR-RR-20. If check fails: reteach by locating and labeling k in each representation explicitly, side by side.

PR-RR-20 Construct genuinely contrasting proportional/non-proportional examples

Mathematical claim: Building a proportional or non-proportional example requires deliberately controlling the origin and rate; two self-built examples meant to contrast must actually differ.

Prerequisites: PR-RR-18

Route Finder

Item: Write one proportional and one non-proportional equation. Verify both against the origin test.

Correct response: Any correct pair, e.g. $y=4x$ (proportional, passes through origin) and $y=4x+1$ (not proportional, does not).

Tripwire

Observed: A student is asked to write a proportional and a non-proportional table, and both tables happen to pass through (0,0) with a constant quotient.

Likely wrong response: Both self-built examples accidentally satisfy the same condition, failing to show real contrast.

Correct response: The non-proportional table must be checked and revised so it genuinely fails the origin or quotient test.

Likely reasoning: *may indicate a self-built example is not being checked against the same test applied to given examples*

Confirm (approach): Give a second construct-and-contrast item in a different representation. If the self-check is skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Build a proportional and a non-proportional graph description for a different context than money or distance. Verify both.

Correct response: Any correctly verified contrasting pair in a new context (e.g., time vs. temperature).

Teach: Require the zero-input check applied to self-built examples, the same way it's applied to given ones.

Fresh Check (Recalculation Check)

Item: Write a proportional and a non-proportional graph description. Verify both.

Correct response: Any correctly verified contrasting pair.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A student is asked to sketch two relationships, one that would work for a subscription-style pricing model and one for a pay-as-you-go model, with no proportionality vocabulary used.

Correct response: Pay-as-you-go (no base fee) is proportional; subscription (with a base fee) is not, since it doesn't pass through (0,0).

Route: If check passes: chapter secure, move to PR-RR-21 (Chapter 15). If check fails: reteach requiring the verification step on self-built work, same as given work.

Chapter 15 -- Scale, Maps, Drawings, and Similar Figures

Chapter Support

Materials: Prepare map strips, real-distance cards, scale-direction arrows, blueprint rectangles, photo enlargement cards, corresponding-side labels, rulers, graph paper, and similarity factor tables.

Differentiation

Students needing concrete support

Use physical strips where one drawing centimetre is paired with the represented real distance. Have students count groups for real-to-drawing division. Use matching side cards before introducing similarity notation.

Students needing language support

Use the frames:

“The scale says 1 _____ represents _____.”

“I am moving from _____ to _____, so I _____.”

“The scale factor is _____.”

“The corresponding sides are _____ and _____.”

“The figures are/are not similar because _____.”

Students ready to extend

Use fractional scales, multi-step map conversions, non-unit scale statements, photo and model enlargements, and the distinction between length scale factor and area scale factor.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given a scale (e.g., 1 cm = 15 km), convert a map distance to a real distance and a real distance to a map distance, and explain why the operations are opposite.
2. Given two figures, check every pair of corresponding sides for a consistent scale factor, and explain what a single-pair check would miss.
3. Create a scaling problem where the additive-shortcut error (adding rather than multiplying) produces a plausible but wrong length.
4. Explain why the scale factor for area is the square of the scale factor for length, using a specific pair of similar figures.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-21 Apply a scale factor in the correct direction

Mathematical claim: Converting map-to-real and real-to-map are inverse operations; direction is determined by which quantity is known and which is requested.

Prerequisites: PR-FR-06

Route Finder

Item: A map uses 1 cm = 8 km. The map shows 5 cm. What is the real distance?

Correct response: 40 km (5×8).

Tripwire

Observed: A map uses 1 cm = 10 km. A real distance of 65 km needs to be shown on the map. A student divides $10 \div 65$.

Likely wrong response: An incorrect small value from dividing in the wrong direction ($10 \div 65$ instead of $65 \div 10$).

Correct response: 6.5 cm ($65 \div 10$).

Likely reasoning: may indicate the scale direction is being reversed -- treating real-to-map as the same operation as map-to-real, or vice versa

Confirm (approach): Give a real-to-map item immediately after a map-to-real item with similar numbers, checking whether direction transfers correctly. If direction is reversed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A blueprint uses 1 inch = 6 feet. The blueprint shows 3.5 inches. What is the real length?

Correct response: 21 feet (3.5×6).

Teach: Scale-direction arrows. Talk Prompt: "Are you moving from drawing to real, or real to drawing? Should the answer be larger or smaller?"

Fresh Check (Recalculation Check)

Item: A map uses 1 cm = 12 km. A real distance is 90 km. How many cm on the map?

Correct response: 7.5 cm.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A model train is built at a scale where 1 cm represents 87 cm of real train, and a real train car is 17.4 m long. How long is the model, asked with no direction-of-conversion hint?

Correct response: 20 cm ($1740 \text{ cm} \div 87$).

Route: If check passes: move to PR-RR-22. If check fails: reteach with the direction arrows drawn explicitly before every conversion.

PR-RR-22 Verify similarity by checking all corresponding side pairs

Mathematical claim: Two figures are similar only if every corresponding side pair shares the same scale factor; checking one pair is insufficient, and different size alone doesn't rule out similarity.

Prerequisites: PR-RR-21

Route Finder

Item: Rectangle A is 4 by 6. Rectangle B is 6 by 9. Are they similar? Check all pairs.

Correct response: Yes -- both pairs scale by 1.5 ($4 \rightarrow 6$, $6 \rightarrow 9$).

Tripwire

Observed: Triangle A has sides 3, 4, 5. Triangle B has sides 6, 8, 9. A student checks $3 \rightarrow 6$ (factor 2) and $4 \rightarrow 8$ (factor 2), concludes similar without checking the third pair.

Likely wrong response: "Similar," from checking only some of the corresponding pairs.

Correct response: Not similar -- $5 \rightarrow 9$ is a factor of 1.8, not 2, so not all corresponding pairs share the same factor.

Likely reasoning: may indicate only one or some corresponding pairs are being checked, with the rest assumed to match

Confirm (approach): Give a second pair of figures where two pairs match the scale factor but one doesn't, disqualifying similarity. If the partial check recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Rectangle A is 6 by 10. Rectangle B is 9 by 14. Are they similar? Check every pair.

Correct response: Not similar -- $6 \rightarrow 9$ is factor 1.5, but $10 \rightarrow 14$ is factor 1.4.

Teach: Side-by-side numerical check required for every corresponding pair, not a visual comparison.

Fresh Check (Recalculation Check)

Item: Rectangle A is 5 by 8. Rectangle B is 10 by 15. Similar? Check all pairs.

Correct response: Not similar -- $5 \rightarrow 10$ is factor 2, but $8 \rightarrow 15$ is factor 1.875.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: Two rectangular picture frames, 8x12 and 12x18 inches, are compared by a framer to see if one is a scaled version of the other, with no 'similar figures' language used.

Correct response: Similar -- both dimensions scale by exactly 1.5.

Route: If check passes: move to PR-RR-23. If check fails: reteach requiring every corresponding pair checked, not just the first.

PR-RR-23 Distinguish scaling a figure from combining measurements

Mathematical claim: Scaling a length or figure by a factor is multiplicative, distinct from combining two separate measurements by addition.

Prerequisites: PR-RR-12

Route Finder

Item: A model is scaled up by a factor of 3. If the model's height is 5 cm, what is the real height?

Correct response: 15 cm (5×3).

Tripwire

Observed: A drawing is scaled by a factor of 4. Two side lengths of 6 cm and 4 cm are given, and a student adds them (10 cm) before scaling instead of scaling each separately.

Likely wrong response: Combines the two lengths additively before or instead of applying the scale factor to each.

Correct response: Each length must be scaled by 4 individually (24 cm and 16 cm), not combined first.

Likely reasoning: may indicate scaling is being confused with combining, applying addition where the scale factor should be multiplied

Confirm (approach): Give a second scaling item where an additive combination produces a plausible but wrong answer. If combining recurs, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A blueprint is scaled by a factor of 8. Two room dimensions are 3.5 m and 2 m. Find both real dimensions.

Correct response: 28 m and 16 m.

Teach: Scale-direction arrows applied explicitly as multiplication to each measurement separately, not addition.

Fresh Check (Recalculation Check)

Item: A figure is scaled by a factor of 2.5. Two sides are 8 cm and 6 cm. Find both scaled lengths.

Correct response: 20 cm and 15 cm.

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: A model is built at 1:20 scale. Two real measurements of 4 m and 2.5 m need model-size equivalents, phrased as a modeler's question with no scaling formula given.

Correct response: 0.2 m and 0.125 m (each divided by 20).

Route: If check passes: chapter secure, move to PR-RR-24 (Chapter 16). If check fails: reteach scaling each measurement separately before any combination.

Chapter 16 -- Mixed Proportional Problem Solving

Chapter Support

Materials: Prepare classification cards, anchor-model cards from Chapters 1–15, part-percent-whole bars, original-change-final bars, ratio tape strips, unit-rate tables, common-unit comparison tables, proportion tables, coordinate grids, scale-direction strips, and zero-origin cards.

Differentiation

Students needing concrete support

Use classification stations with only two or three choices at first. Require students to build the anchor model before using symbols. For a mixed problem, students can place the problem card beside a model card and explain the match orally.

Students needing language support

Use sentence frames:

“This problem is about _____ because _____.”

“The whole/base is _____.”

“The anchor model is _____.”

“I will use _____ because _____.”

“My answer is reasonable because _____.”

Students ready to extend

Ask students to design pairs of problems that use similar numbers but require different methods. Ask them to create a non-proportional situation that looks proportional at first, then explain the zero-origin or starting-amount evidence.

Strategy Studio

These tasks deliberately vary the numbers, representation, unknown, or context. Do not choose a method from the chapter title alone.

1. Given four short scenarios (one each involving percent, ratio, rate, and scale), classify each by anchor model before solving any of them.
2. Create a mixed problem that superficially resembles a percent problem but is actually a ratio problem, and explain what signals the correct classification.
3. Solve a correctly classified problem, then verify it using that anchor model's own established check from its home chapter.
4. Explain, using a specific example, why classification must happen before calculation rather than after.

Talk, Compare, Revise

Work with a partner. Each person explains one strategy. Identify one feature both strategies preserve, one feature that differs, and one sentence you would revise to make the reasoning clearer.

Student Self-Check

Before the exit ticket, complete these sentences: I identified the relationship by _____. The representation that made the structure clearest was _____. My answer is reasonable because _____.

PR-RR-24 Classify the anchor model before calculating

Mathematical claim: A mixed problem must be classified by which relationship it shows (percent, ratio, rate, scale, or proportionality) before any calculation begins.

Prerequisites: PR-RR-01, PR-PC-07, PR-RR-04, PR-RR-21, PR-RR-18

Route Finder

Item: “A jacket is 20% off.” Which anchor model applies, before calculating anything?

Correct response: Percent (specifically, percent decrease).

Tripwire

Observed: Given “Boys:girls = 3:4, total 28,” a student immediately sets up a percent calculation (28×0.75) without first classifying this as a part-to-part ratio problem.

Likely wrong response: Applies a percent procedure to a ratio problem, or any mismatched procedure applied before classification.

Correct response: This is a ratio problem (part-to-part-to-whole), requiring the total-parts method, not a percent calculation.

Likely reasoning: *may indicate a familiar procedure is being applied immediately without first classifying which anchor model actually fits -- if this recurs, check which specific earlier skill (PR-RR-01, PR-PC-07, PR-RR-04, PR-RR-21, or PR-RR-18) the misclassification traces back to*

Confirm (approach): Give a second set of superficially similar problems, each requiring a different anchor model, checking whether classification precedes calculation. If misclassification recurs, treat as confirmed and route to the specific earlier skill it traces to.

Confirmation Item

Same claim, different surface form.

Item: “A scale drawing uses 1 cm = 3 km, and the drawing shows 7 cm.” Classify which anchor model applies before calculating.

Correct response: Scale (map/drawing to real conversion).

Teach: Five-step decision routine (name, identify, choose, calculate, check). Talk Prompt: “What relationship do the quantities show, and which anchor model makes that visible?”

Fresh Check (Recalculation Check)

Item: “A taxi charges a \$4 starting fee plus \$2 per km.” Classify before calculating.

Correct response: Proportionality/rate check -- specifically, this is NOT proportional (nonzero starting fee).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: “A phone plan costs \$20 plus \$0.10 per text.” A customer asks whether the cost is proportional to number of texts, with no mention of anchor models or classification.

Correct response: Rate/proportionality check -- not proportional (nonzero cost at 0 texts).

Route: If check passes: move to PR-RR-25. If check fails: route to the specific earlier chapter’s skill matching the misclassified anchor model, not a generic review.

PR-RR-25 Solve and verify a mixed problem once classified

Mathematical claim: Once classified, a mixed problem is solved using that anchor model's own established method and verified the same way that method is verified elsewhere in the book.

Prerequisites: PR-RR-24

Route Finder

Item: Classified as a ratio problem: boys:girls=3:4, total 28. Solve for the number of boys.

Correct response: 12 boys ($\frac{3}{7}$ of 28).

Tripwire

Observed: A problem is correctly classified as a proportion (cross-products), but a student sets up the proportion with the terms reversed on one side.

Likely wrong response: An incorrect equation from reversed terms, despite correct classification.

Correct response: Correct setup requires matching which quantity is the numerator on both sides, consistent with the classified anchor model's own method (PR-RR-15).

Likely reasoning: *may indicate the anchor model was correctly identified but its own specific execution method (e.g., matching numerator roles) was not applied correctly -- a different gap from misclassification*

Confirm (approach): Give a second correctly-classified item testing the same anchor model's specific known error. If the execution error recurs with correct classification, treat as confirmed and route to that anchor model's home skill.

Confirmation Item

Same claim, different surface form.

Item: Classified as a percent problem: a \$60 item is discounted 25%. Solve and verify.

Correct response: \$45; verify by substituting back ($60 \times 0.75 = 45$).

Teach: Apply the named anchor model's established check from its home chapter (e.g., PR-RR-15's diagonal-pairing check for cross-products).

Fresh Check (Recalculation Check)

Item: Classified as a scale problem: map scale 1cm=20km, map shows 4.5cm. Solve and verify.

Correct response: 90 km; verify by dividing back ($90 \div 20 = 4.5$).

Road Test

Less-cued, mixed context -- tests whether the skill transfers without being told which chapter this is from.

Item: “A fruit stand sells apples at 3 for \$2, and a customer buys 12 apples,” asked as a plain word problem with no classification instruction given.

Correct response: \$8 (unit rate $\$ \frac{2}{3}$ per apple $\times$ 12, or scale factor 4 applied to \$2).

Route: Chapter 16 and the full Book 3 skill map are complete once both PR-RR-24 and PR-RR-25 are secure. No further chapter to route to within this book.